

Task Resource Booklet

JCT Mathematics

An tSraith Shóisearach do Mhúinteoirí

Junior **CYCLE**
for teachers



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Department of Education



This Task Resource Booklet contains a selection of mathematical task and activities which have been engaged with by teachers for the purposes of professional development.

The tasks are suitable for use with students of junior cycle mathematics.

Teachers should use their professional judgement when considering the suitability of a mathematical task or activity and should adapt the task, if necessary, to meet the needs and/or context of their students.

Tasks have been designed to be accessible, challenging and extendable. Adaptations should seek to maintain these design principles.

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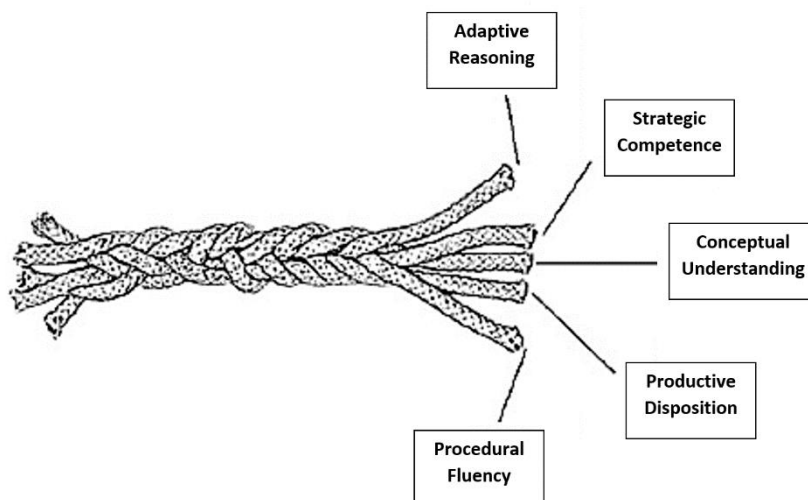
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The Aim of Junior Cycle Mathematics

The aim of junior cycle mathematics is to provide relevant and challenging opportunities for all students to become mathematically proficient so that they can cope with the mathematical challenges of daily life and enable them to continue their study of mathematics in senior cycle and beyond. In this specification, mathematical proficiency is conceptualised not as a one-dimensional trait but as having five interconnected and interwoven components:

- conceptual understanding—comprehension of mathematical concepts, operations, and relations
- procedural fluency—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- strategic competence—ability to formulate, represent, and solve mathematical problems in both familiar and unfamiliar contexts
- adaptive reasoning—capacity for logical thought, reflection, explanation, justification and communication
- productive disposition—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence, perseverance and one’s own efficacy.

Mathematics Specification pg. 5



National Research Council (2001) Adding it up: *Helping children learn mathematics*.

J. Kilpatrick, J. Swafford, and B. Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.

Action Verbs

Learning outcomes are statements that describe the knowledge, understanding, skills and values students should be able to demonstrate after a period of learning. In order to fully support the exploration of the learning outcomes by teachers, parents, and students a glossary of the action verbs used in the specification (pg. 25) is included below:

Action verbs	Students should be able to
Analyse	study or examine something in detail, break down to bring out the essential elements or structure; identify parts and relationships, and to interpret information to reach conclusions
Apply	select and use knowledge and/or skills to solve a problem in a new situation
Calculate	work out a numerical answer
Classify	group things based on common characteristics
Compare	give an account of the similarities and (or) differences between two (or more) items or situations, referring to both (all) of them throughout
Construct	use properties of shapes and geometric results to draw accurately, using only the prescribed geometrical tools
Convert	change from one form to another
Define	[a set]: give a rule that identifies the elements of a set
Discuss	offer a considered, balanced review that includes a range of arguments, factors or hypotheses; opinions or conclusions should be presented
Estimate	state or calculate a rough value for a particular quantity
Evaluate	judge the relative quality or validity of something, which may include analysing, comparing and contrasting, criticising, defending, or judging
Explain	give a reasoned account, showing how causes lead to outcomes
Generalise	generate a general statement based on specific instances
Generate	Produce or create
Interpret	use knowledge and understanding to explain the meaning of something in context

Investigate	observe, study, or make a detailed and systematic examination to establish facts and reach new conclusions
Justify	give valid reasons or evidence to support an answer or conclusion
Mathematise	generate a mathematical representation (e.g. graph, equation, geometric figure) to describe a particular aspect of a phenomenon
Prove	give a deductive argument to demonstrate that a particular statement is true, including reasons for each step in the argument
Round	give the number in the required form (for example, a multiple of 100, or a number with three significant figures) that is closest in absolute terms to a particular number
Sketch	draw a rough diagram or graph without using geometrical tools
Solve	work out an answer or solution to
State	provide a concise statement with little or no supporting argument
Understand	have detailed knowledge of, be able to use appropriately, and see the connections between parts
Use	apply knowledge or rules to put theory into practice
Verify	demonstrate that a statement is true

Action Verb Matching Activity 1

Complete the following activity by matching the action verb to the correct statement.

Interpret	Prove	Understand	Round	Calculate	Mathematise	Compare
State	Construct	Explain	Discuss	Verify	Convert	Use

Action Verb	Students should be able to
	work out a numerical answer
	use properties of shapes and geometric results to draw accurately, using only the prescribed geometrical tools
	change from one form to another
	give the number in the required form (for example a multiple of 100, or a number with three significant figures) that is closest in absolute terms to a particular number
	use knowledge and understanding to explain the meaning of something in context
	have detailed knowledge of, be able to use appropriately, and see the connections between parts
	demonstrate that a statement is true
	give a deductive argument to demonstrate that a particular statement is true, including reasons for each step in the argument
	offer a considered, balanced review that includes a range of arguments, factors or hypotheses; opinions or conclusions should be presented clearly and supported by appropriate evidence
	give an account of the similarities and (or) differences between two (or more) items or situations, referring to both (all) of them throughout
	generate a mathematical representation (e.g. graph, equation, geometric figure) to describe a particular aspect of a phenomenon
	give a reasoned account, showing how causes lead to outcomes
	provide a concise statement with little or no supporting argument
	apply knowledge or rules to put theory into practice

Action Verb Matching Activity 2

Complete the following activity by matching the action verb to the correct statement.

Define	Analyse	Classify	Evaluate	Apply	Investigate
Solve	Justify	Sketch	Estimate	Generalise	Generate

Action Verb	Students should be able to
	state or calculate a rough value for a particular quantity
	[a set]: give a rule that identifies the elements of a set
	work out an answer or solution to
	select and use knowledge and/or skills to solve a problem in a new situation
	give valid reasons or evidence to support an answer or conclusion
	judge the relative quality or validity of something, which may include analysing, comparing and contrasting, criticising, defending or judging
	observe, study or make a detailed and systematic examination to establish facts and reach new conclusions
	draw a rough diagram or graph without using geometrical tools
	study or examine something in detail, break down to bring out the essential elements or structure; identify parts and relationships, and to interpret information to reach conclusions
	group things based on common characteristics
	produce or create
	generate a general statement based on specific instances

Unifying Strand

This strand permeates all the four contextual strands (Number, Geometry & Trigonometry, Algebra & Functions, and Statistics & Probability) and is composed of the six elements of the specification, which are shown below.

There is no specific content linked to this strand; rather, its learning outcomes underpin the rest of the specification. Each learning outcome in this strand is applicable to all of the activities and content of the other four strands—for example, students should be able to draw on all of their mathematical knowledge and skills to solve a problem or to communicate mathematics.

Furthermore, the elements of this strand are interdependent, so that students should develop the different skills associated with each element in tandem rather than in isolation - for example, engaging in problem-solving can help students improve their understanding of building blocks and their ability to make connections within mathematics.

The Elements

Element	
<i>Building Blocks</i>	Students should understand and recall the concepts that underpin each strand and be able to carry out the resulting procedures accurately, effectively, and appropriately.
<i>Representation</i>	Students should be able to represent a mathematical situation in a variety of different ways and translate flexibly between them.
<i>Connections</i>	Students should be able to make connections within strands and between strands, as well as connections between mathematics and the real world.
<i>Problem-Solving</i>	Students should be able to investigate patterns, formulate conjectures, and engage in tasks in which the solution is not immediately obvious, in familiar and unfamiliar contexts.
<i>Generalisation and Proof</i>	Students should be able to move from specific instances to general mathematical statements, and to present and evaluate mathematical arguments and proofs.
<i>Communication</i>	Students should be able to communicate mathematics effectively in verbal and written form.

Unifying Strand

Element: Building blocks

Students should be able to:

- U.1 recall and demonstrate understanding of the fundamental concepts and procedures that underpin each strand
- U.2 apply the procedures associated with each strand accurately, effectively, and appropriately
- U.3 recognise that equality is a relationship in which two mathematical expressions have the same value

Element: Representation

Students should be able to:

- U.4 represent a mathematical situation in a variety of different ways, including: numerically, algebraically, graphically, physically, in words; and to interpret, analyse, and compare such representations

Element: Connections

Students should be able to:

- U.5 make connections within and between strands
- U.6 make connections between mathematics and the real world

Element: Problem solving

Students should be able to:

- U.7 make sense of a given problem, and if necessary mathematise a situation
- U.8 apply their knowledge and skills to solve a problem, including decomposing it into manageable parts and/or simplifying it using appropriate assumptions
- U.9 interpret their solution to a problem in terms of the original question
- U.10 evaluate different possible solutions to a problem, including evaluating the reasonableness of the solutions, and exploring possible improvements and/or limitations of the solutions (if any)

Element: Generalisation and proof

Students should be able to:

- U.11 generate general mathematical statements or conjectures based on specific instances
- U.12 generate and evaluate mathematical arguments and proofs

Element: Communication

Students should be able to:

- U.13 communicate mathematics effectively: justify their reasoning, interpret their results, explain their conclusions, and use the language and notation of mathematics to express mathematical ideas precisely

Webinar Tasks

Triangle Sort Card Task

This task can be used with students at the beginning, during or at the end of their initial study of triangles. If the task is used at the beginning or during a unit of learning the task is formative as it provides an opportunity for the class teacher to assess the level of knowledge of their students regarding the identification, sorting, organising and categorisation of various triangles, and adapt the focus of learning.

The Primary Curriculum identifies the following areas that incoming first years will have engaged with:

- Make informal deductions about 2-D shapes and their properties
- Use angle and line properties to describe and classify triangles
- Plot simple coordinates and apply where appropriate
- Use 2-D shapes and their properties to solve problems
- Explore the sum of the angles in a triangle

The task allows for the reinforcement of students' knowledge and understanding from primary school, while simultaneously providing an opportunity for extension and enrichment for students who may have fully grasped the concepts before commencing first year in post-primary. To promote students' mathematical thinking and discussion, and to generate rich classroom dialogue it is recommended that the task be undertaken by students in groups of between 2 and 4 students.

Suggested instructions for using this task:

- Each group should be given a set of Triangle Card Sort cards (A4) and a classification grid (A3)
- In turn, students in the group should select a card. The student identifies where they believe the card should be placed on the grid
- The card can only be placed on the grid once the group has reached a consensus
- The group can postpone the placement of a card only twice during the task and only after a discussion has taken place about its placement. [This task condition is to ensure students engage with some of the more challenging cards. The class teacher is best placed to decide whether this task condition is appropriate]
- The teacher should move around the room informally gathering information (evidence) about student's knowledge and learning. Appropriate questioning often provides greater insight
- Once the task has been completed, a plenary discussion is recommended. This should be informed by information (evidence) gathering process and where appropriate questioning has the potential to enrich the class discussion and student learning.

The task is linked to the following contextual strand learning outcomes from the Junior Cycle Mathematics specification:

- GT2 investigate 2-D shapes
- GT3 investigate the concept of proof through their engagement with geometry
 - Theorem 1,2,4,8,14
- GT5 investigate properties of points, lines and line segments in the co-ordinate plane

The task is also linked to the following Unifying strand learning outcomes from the Junior Cycle Mathematics specification:

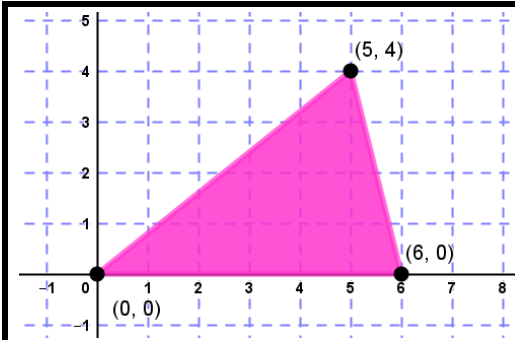
- U1 recall and demonstrate understanding of the fundamental concepts and procedures that underpin each strand
- U5 make connections within and between strands
- U13 communicate mathematics effectively: justify their reasoning, interpret their results, explain their conclusions, and use the language and notation of mathematics to express mathematical ideas precisely

The task develops students' understanding of connections within the Geometry and Trigonometry strand and can be used to strengthen students' understanding of the connections between synthetic and co-ordinate geometry. It is suggested that this should be a focus of the plenary. It may be necessary to reduce the difficulty level of the task by selectively removing some of the cards. It is recommended, however, that scaffolding be kept to a minimum to allow students to apply their knowledge in unfamiliar situations and create a need for the construction of new knowledge. If the task is scaffolded students may engage with it multiple times over the three years of Junior Cycle.

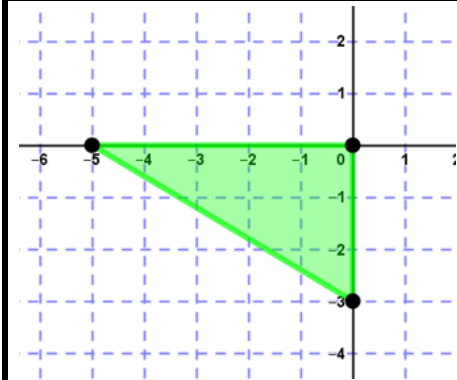
Students should be encouraged to explain and justify their reasoning in relation to where they have placed their cards. Reasoning is often done verbally, and accurate use of mathematical language should be developed. Students should also be encouraged to use geometrical tools (compass etc.) to justify their placement where appropriate. Where appropriate, students should be alerted to the difference between justification and formal proof.

Teachers can encourage student talk using effective questioning and active listening. Below are some examples of questions that reinforce a student's prior knowledge and/or challenge their misconceptions. These questions may also provide an opportunity for extension and enrichment for students who may have fully grasped the concepts in primary school.

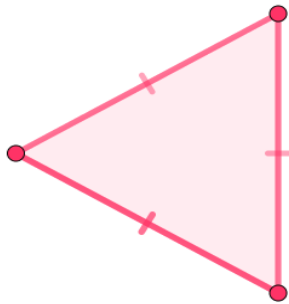
Sample Questions (This is not an exhaustive list)



Change the position of one of the vertices to create a different triangle, but the area of the new triangle must stay the same as the original.



What is the co-ordinate of this vertex? [Referring to $(-5, 0)$] What is the length of this side of the triangle? Discuss.



Construct an equilateral triangle of any size. Explain the process and outline why the triangle is equilateral as a result.

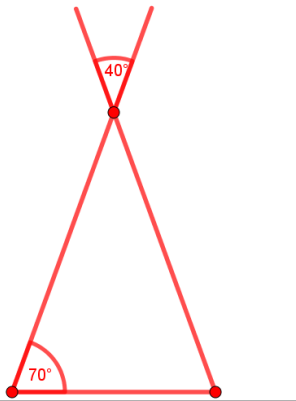
vertices
(1,3), (6,3), (-4,3)

Change any of the co-ordinates to make a triangle?

Change one of the co-ordinates to make an isosceles triangle.
Generalise this change to express it for all possible isosceles triangles given these co-ordinates?

vertices
(0,0) (5,0) (0,5)

Confirm/prove, in as many ways as possible, that this triangle is right angled?



Create a new question. The angles can be changed, but the triangle must remain isosceles.

Task Instructions

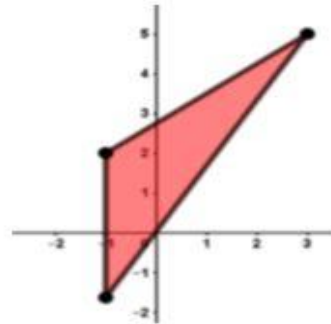
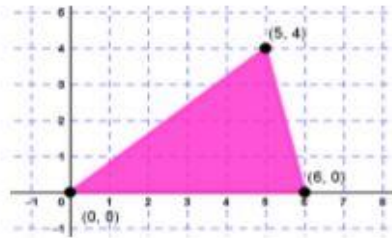
Place each card under the heading
that best describes the information given on the card

Scalene Triangle	Right-angled Triangle	Isosceles Triangle	Equilateral Triangle	Not a Triangle

vertices
(0,0) (6,0) (5,4)

side lengths
3, 4, 6

angle measure
 $91^\circ, 62^\circ, 27^\circ$



angle measure
 $90^\circ 37.36^\circ, 52.64^\circ$

side lengths
3, 4, 5

angle measure
 $60^\circ, 30^\circ, 90^\circ$

vertices
(0,0) (5,0) (0,10)

side lengths
1, 1, $\sqrt{2}$



vertices
(0,0) (5,0) (0,5)

side lengths
3, 3, 4

side lengths
32.6, 32.6, 60



side lengths
5, 12, 13

side lengths
7, 24, 25

side lengths
1, 2, 4

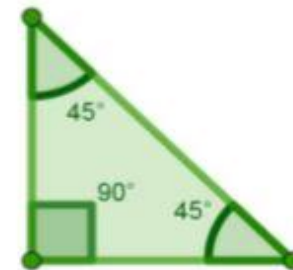
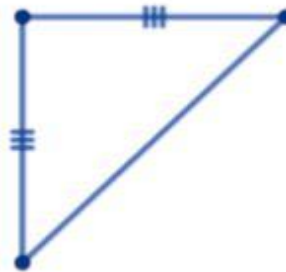
side lengths
11, 12, 25

side lengths
3, 3, 8

angle measure
 $90^\circ, 60^\circ, 60^\circ$

angle measure
 $70^\circ, 70^\circ, 70^\circ$

angle measure
 $90^\circ, 90^\circ, 45^\circ$



vertices
 $(1,2)$ $(1,6)$ $(1,8)$

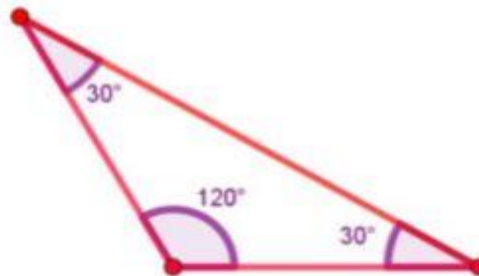
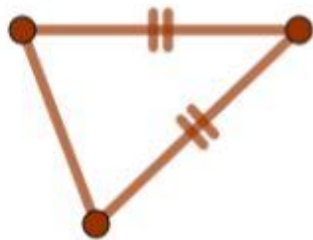
vertices
 $(1,3)$, $(6,3)$, $(-4,3)$

angle measure
 30° , 40° , 50°

side lengths
 $1, 1, 1$

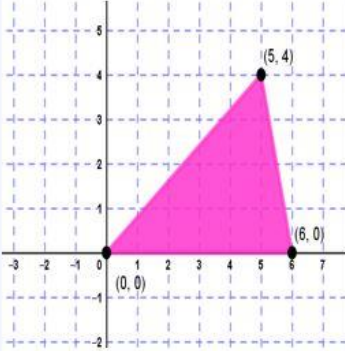
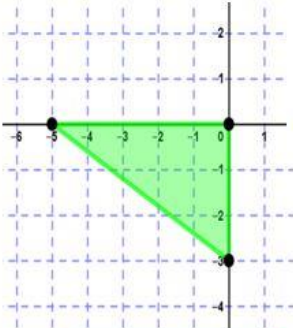
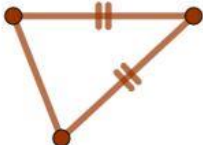
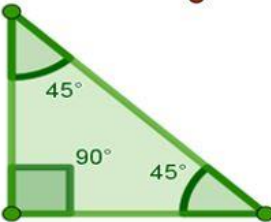
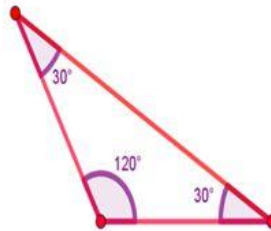
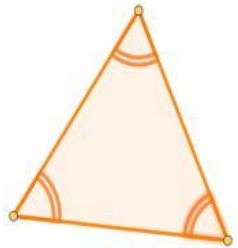

angle measure
 60° 60° 60°

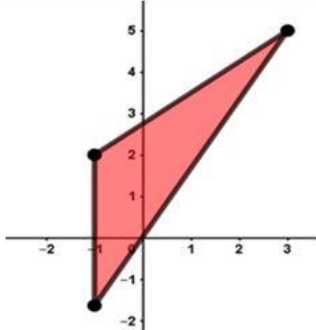
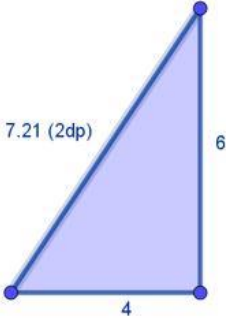
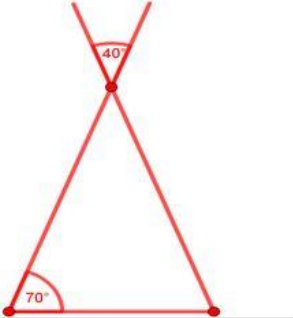
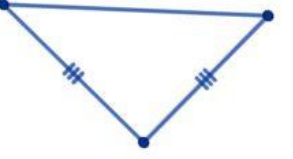
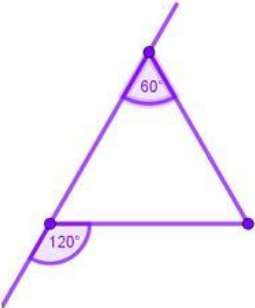
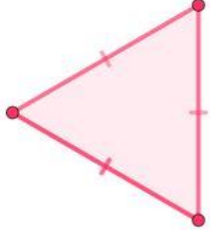
side lengths
 $3, 3, 3$



side lengths
 $19, 180, 181$

Suggested Solution:

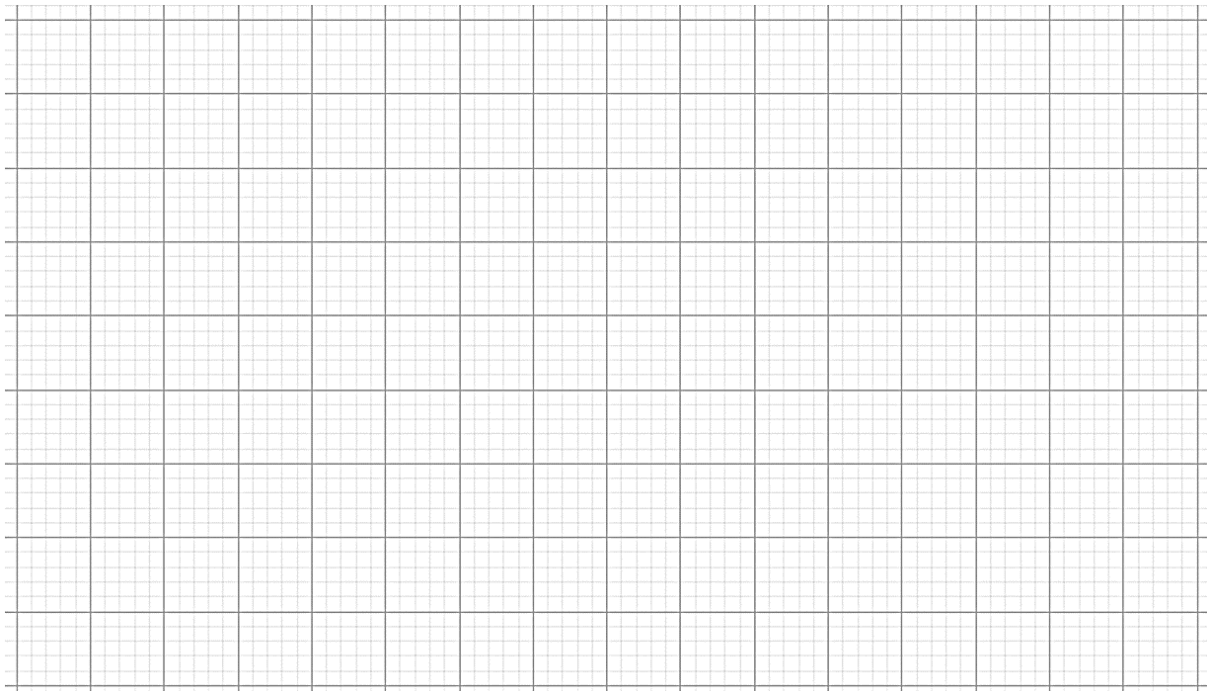
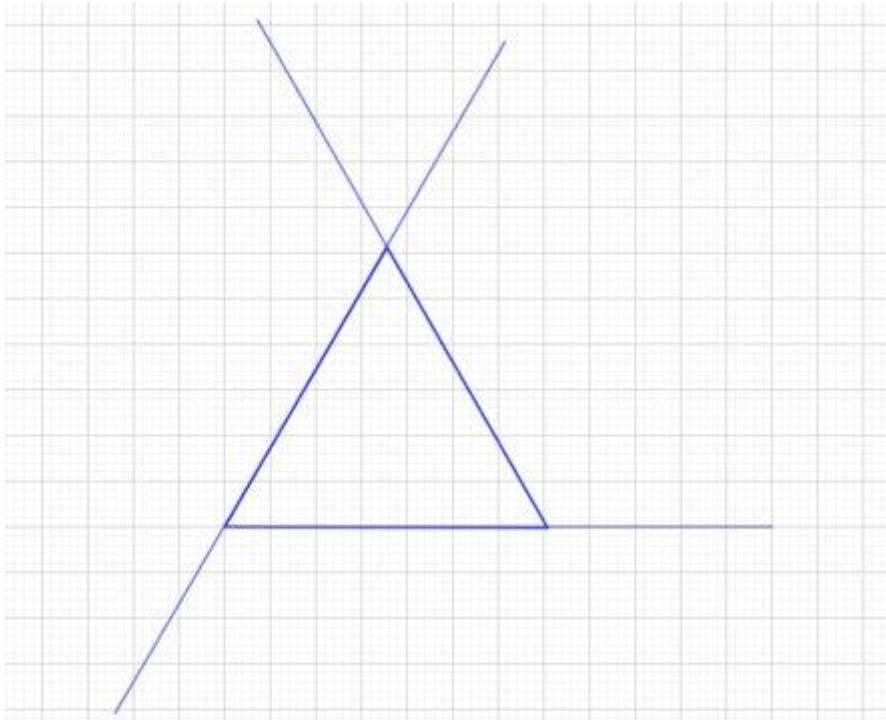
Scalene triangle	Right angled triangle	Isosceles triangle	Equilateral triangle	Not a triangle
<p>side lengths 3, 4, 6</p>  <p>Vertices (0,0), (6,0), (5,4)</p> <p>angle measure 91°, 62°, 27°</p>	<p>angle measure 90°, 37.36°, 52.64°</p> <p>side lengths 3, 4, 5</p> <p>Vertices (0,0), (5,0), (0,10)</p>  <p>side lengths 19, 180, 181</p>	<p>side lengths 3, 3, 4</p>   	<p>side lengths 1, 1, 1</p> <p>angle measure 60°, 60°, 60°</p> 	<p>side lengths 1, 2, 4</p> <p>angle measure 90°, 90°, 45°</p> <p>side lengths 3, 3, 8</p>  <p>angle measure 90°, 60°, 60°</p>

Scalene triangle	Right angled triangle	Isosceles triangle	Equilateral triangle	Not a triangle
	<p style="text-align: center;">Vertices (0,0), (5,0), (0,5)</p> <p>side lengths 5, 12, 13</p> <p>side lengths 1, 1, $\sqrt{2}$</p> <p>angle measure $60^\circ, 30^\circ, 90^\circ$</p>  <p>side lengths 7, 24, 25</p>	<p>side lengths 32.6, 32.6, 60</p>  	  <p>side lengths 3, 3, 3</p>	<p>angle measure $70^\circ, 70^\circ, 70^\circ$</p> <p>side lengths 11, 12, 25</p> <p>Vertices (1,2), (1,6), (1,8)</p> <p>angle measure $30^\circ, 40^\circ, 50^\circ$</p> <p>Vertices (1,3), (6,3), (-4,3)</p>

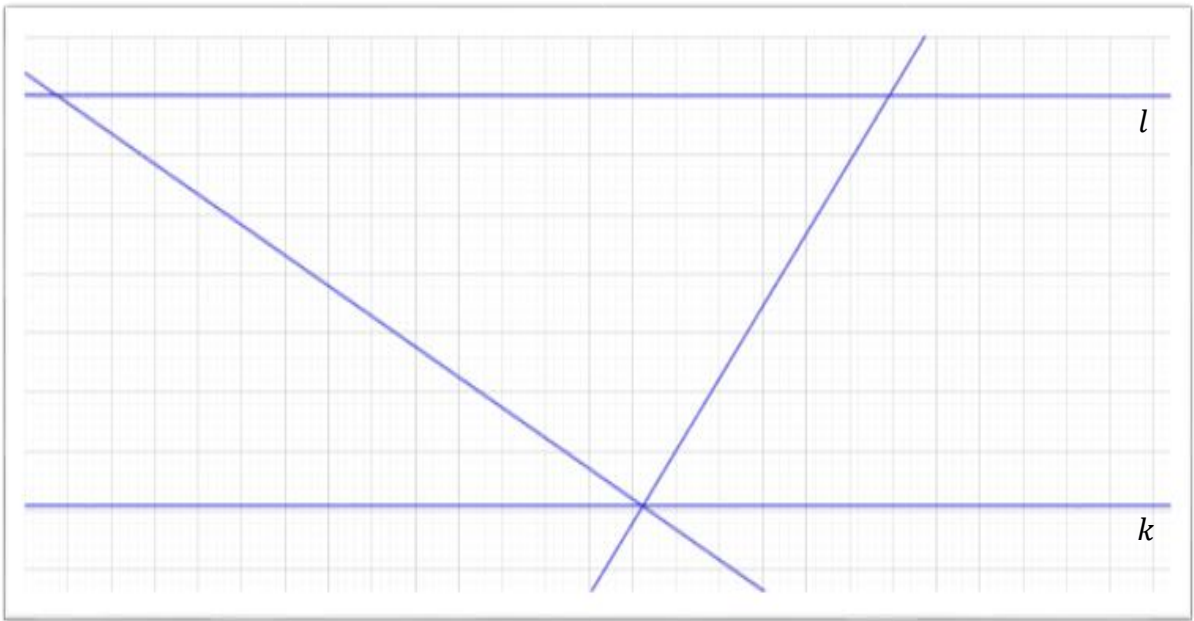
Relationships Tasks

These tasks develop students' understanding of representations within the Unifying Strand. It encourages students to generate and evaluate mathematical arguments and proofs. It may help determine if students are able to identify, and articulate invariant relationships associated with triangles.

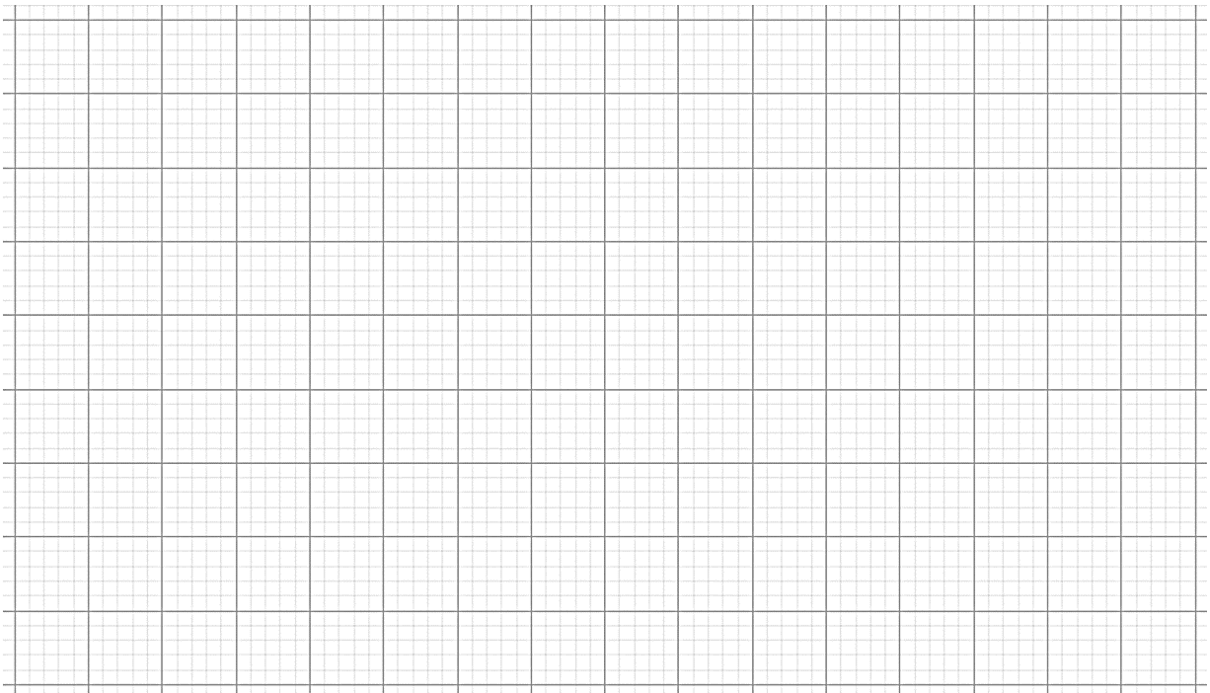
(a) What relationships can you identify?



(b) What relationships can you identify?

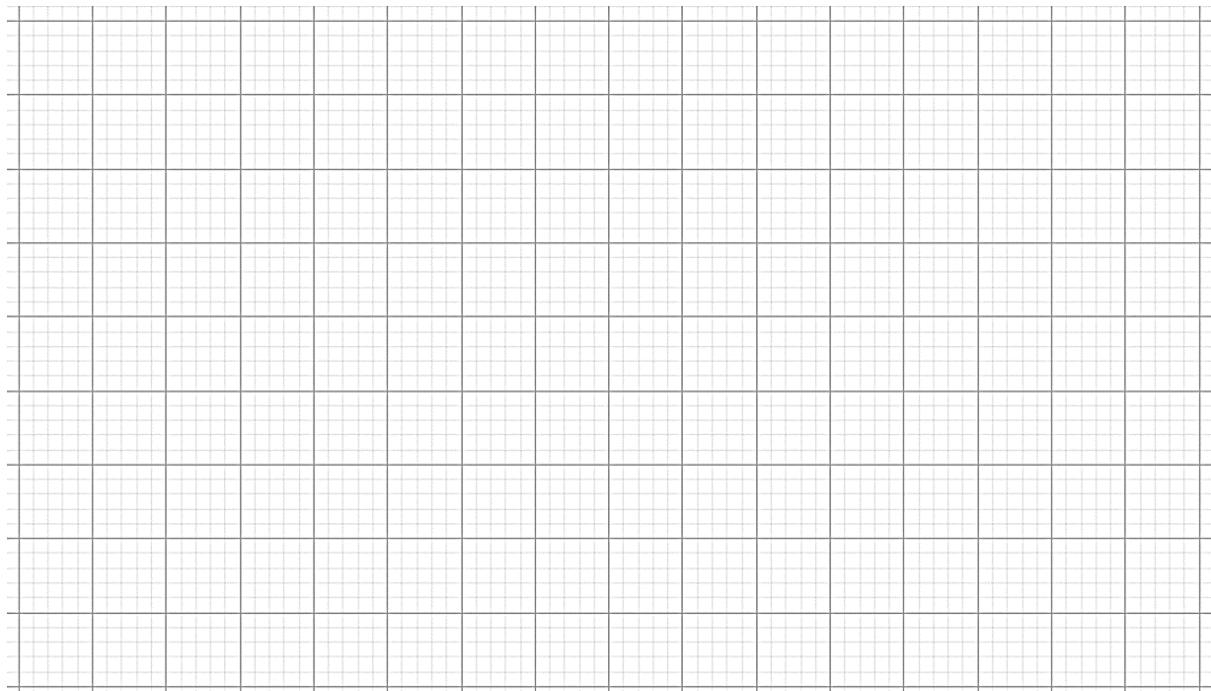
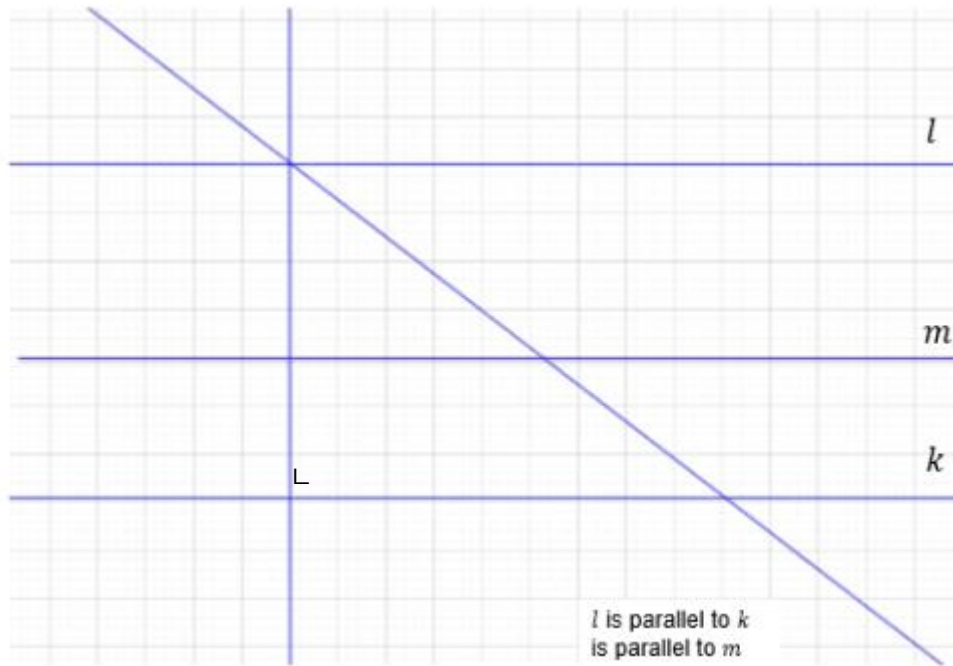


l is parallel to k



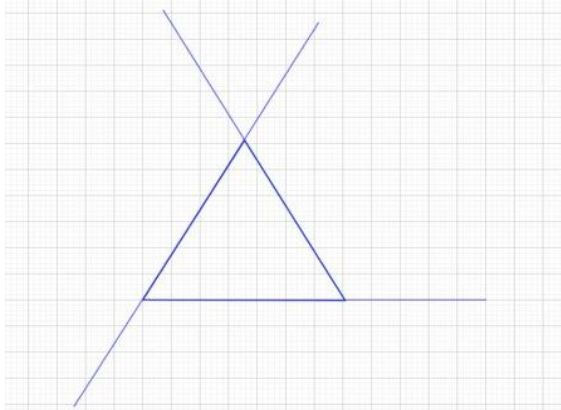
(c) How many relationships can you identify in the diagram below?

- 10
- 11
- 13
- 15



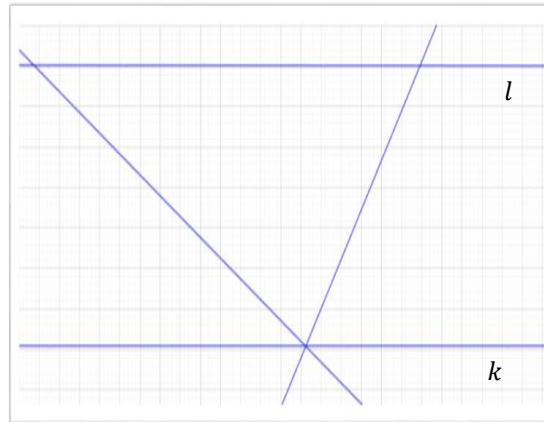
Relationships Tasks Possible Solutions

(a) What relationships can you identify?



- Straight angle has 180°
- Vertically opposite angles are equal
- The sum of the angles in a triangle is equal to 180°
- External angle theorem

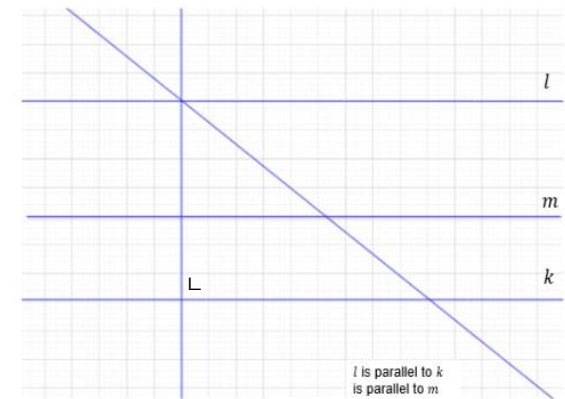
(b) What relationships can you identify?



l is parallel to k

- Straight angle has 180°
- Vertically opposite angles are equal
- The sum of the angles in a triangle is equal to 180°
- External angle theorem
- Corresponding angles are equal
- Alternate angles are equal
- Perpendicular distance between parallel lines
- Two sides of a triangle are together greater than the third
- Largest angle is across from the longest side

(c) How many relationships can you identify in the diagram below?



11 relationships –

- Straight angle has 180°
- Vertically opposite angles are equal
- The sum of the angles in a triangle is equal to 180°
- External angle theorem
- Perpendicular distance between parallel lines
- Two sides of a triangle are together greater than the third
- Corresponding angles are equal
- Alternate angles are equal
- Largest angle is across from the longest side
- Similar triangles
- Scaling and scale fact

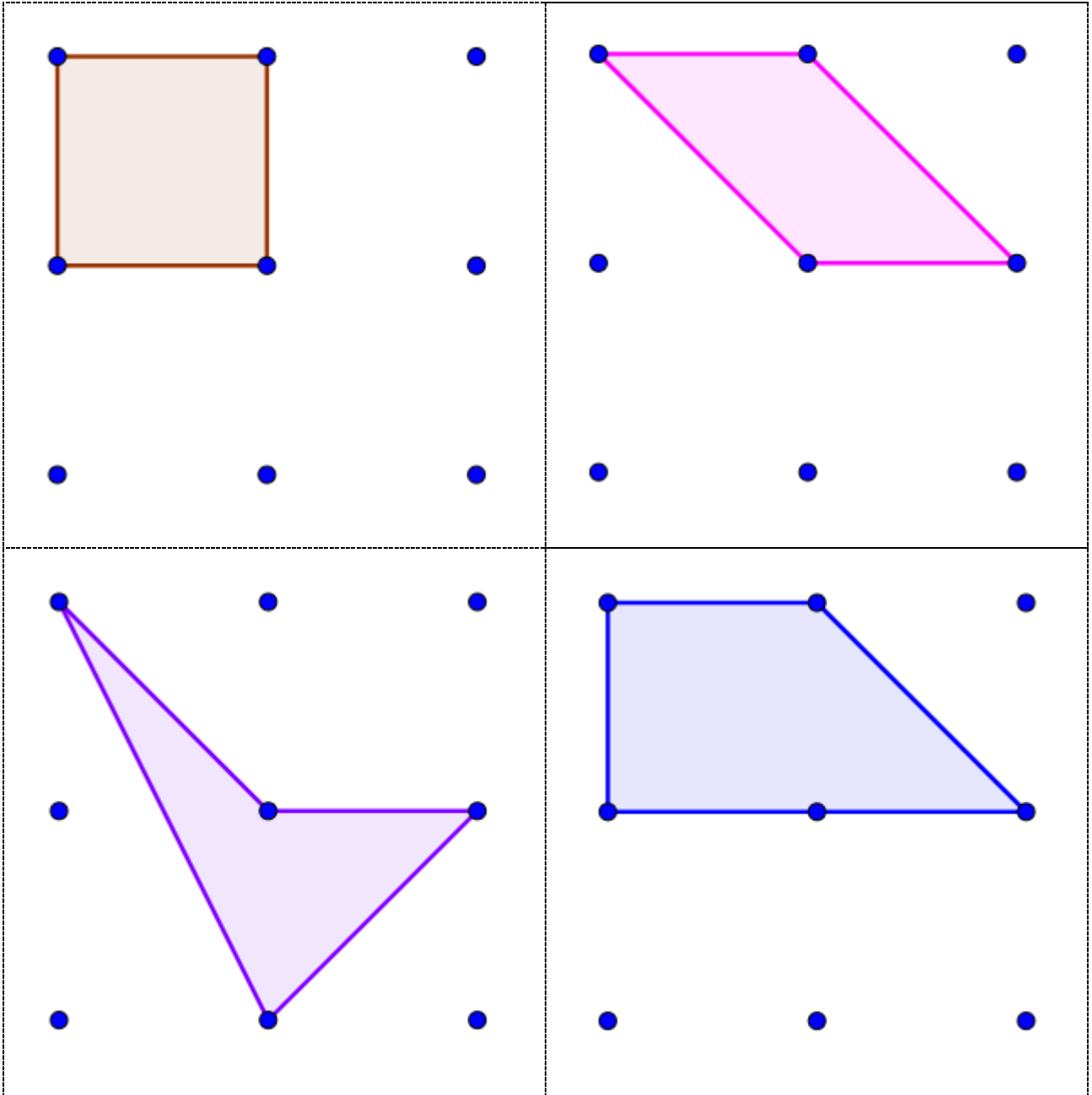
Symposium Tasks

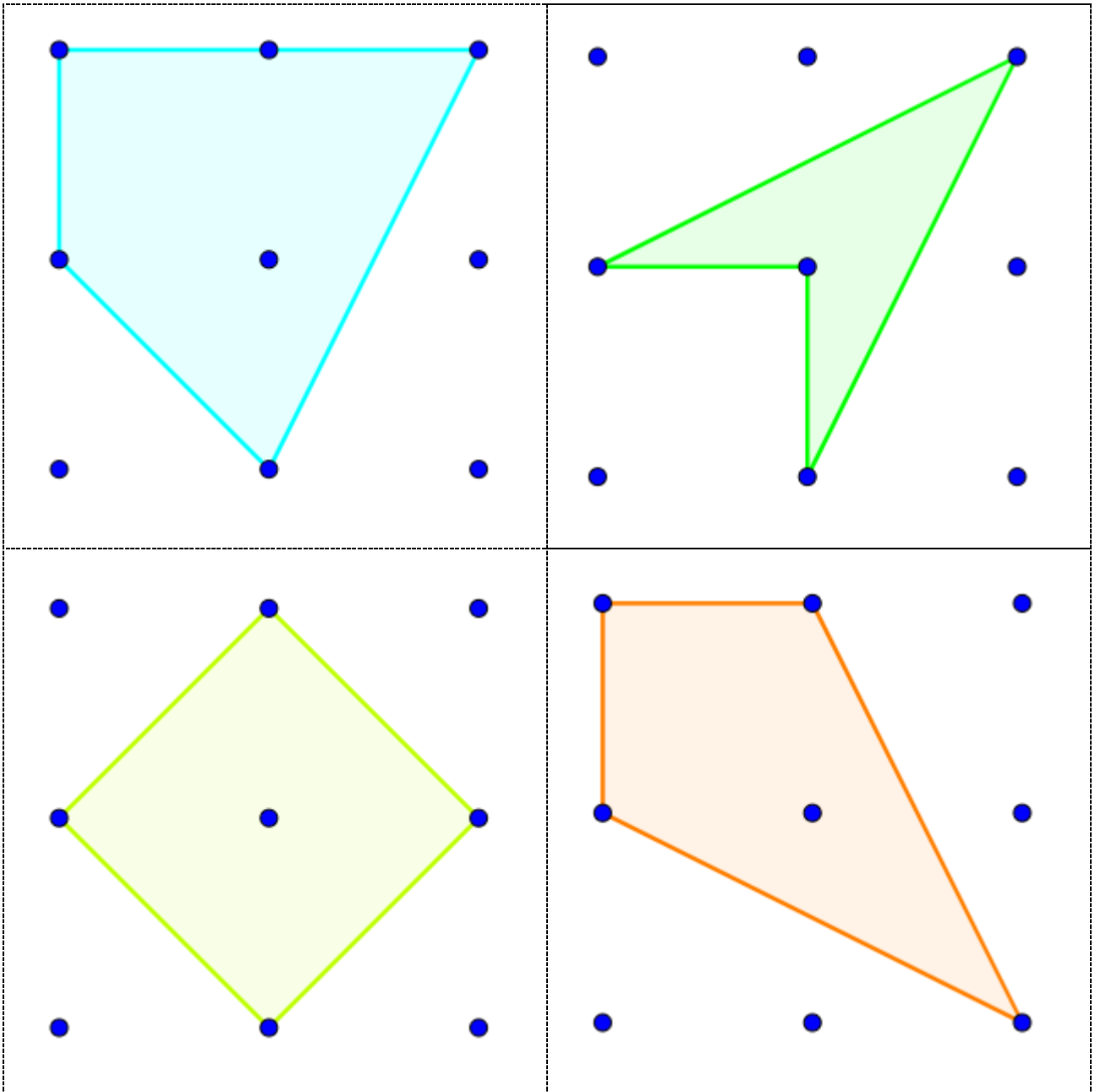
Classifying Quadrilaterals

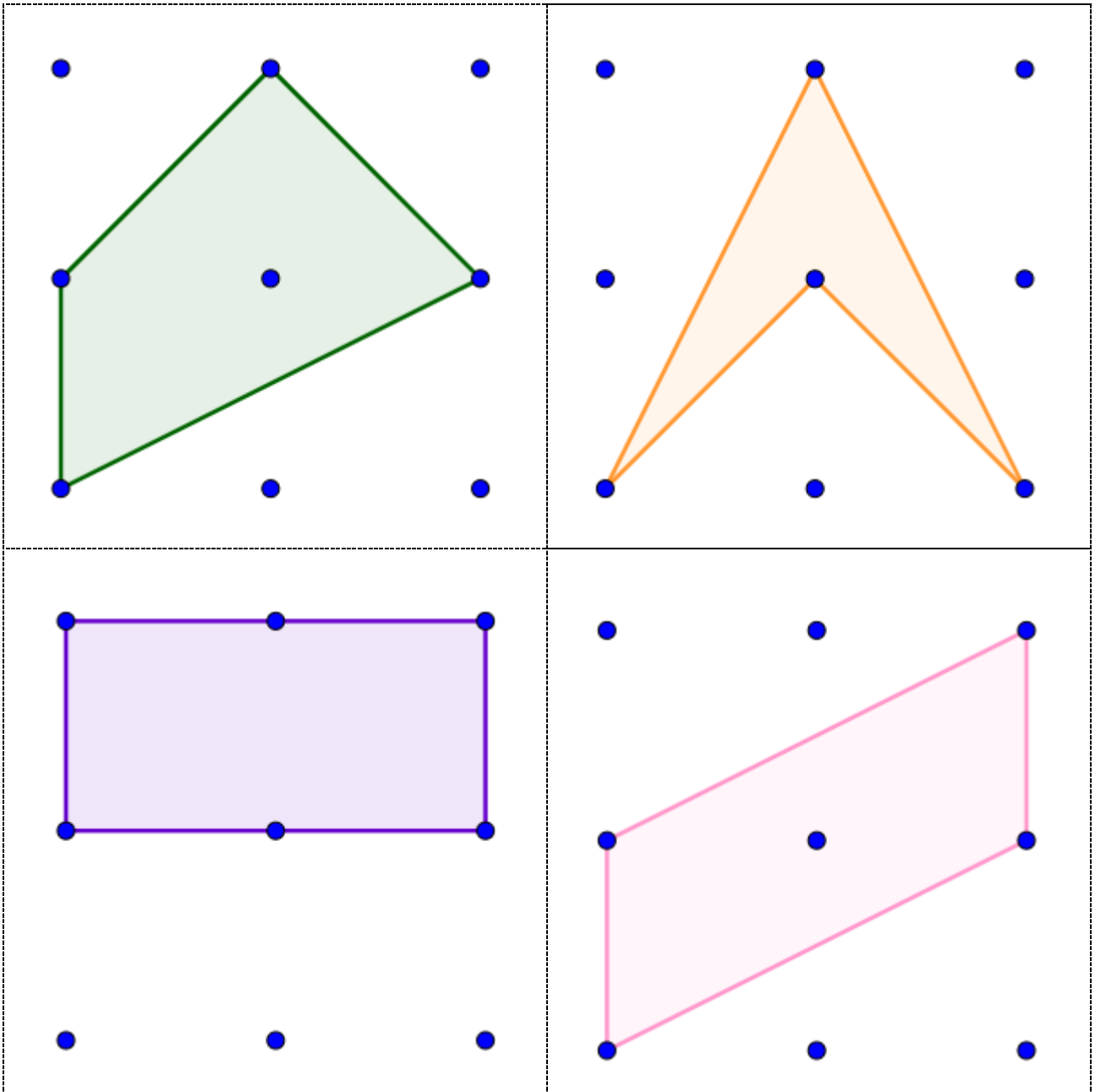
To promote students' mathematical thinking and discussion, and to generate rich classroom dialogue, it is recommended that the task be undertaken in groups of between 2 and 4 students.

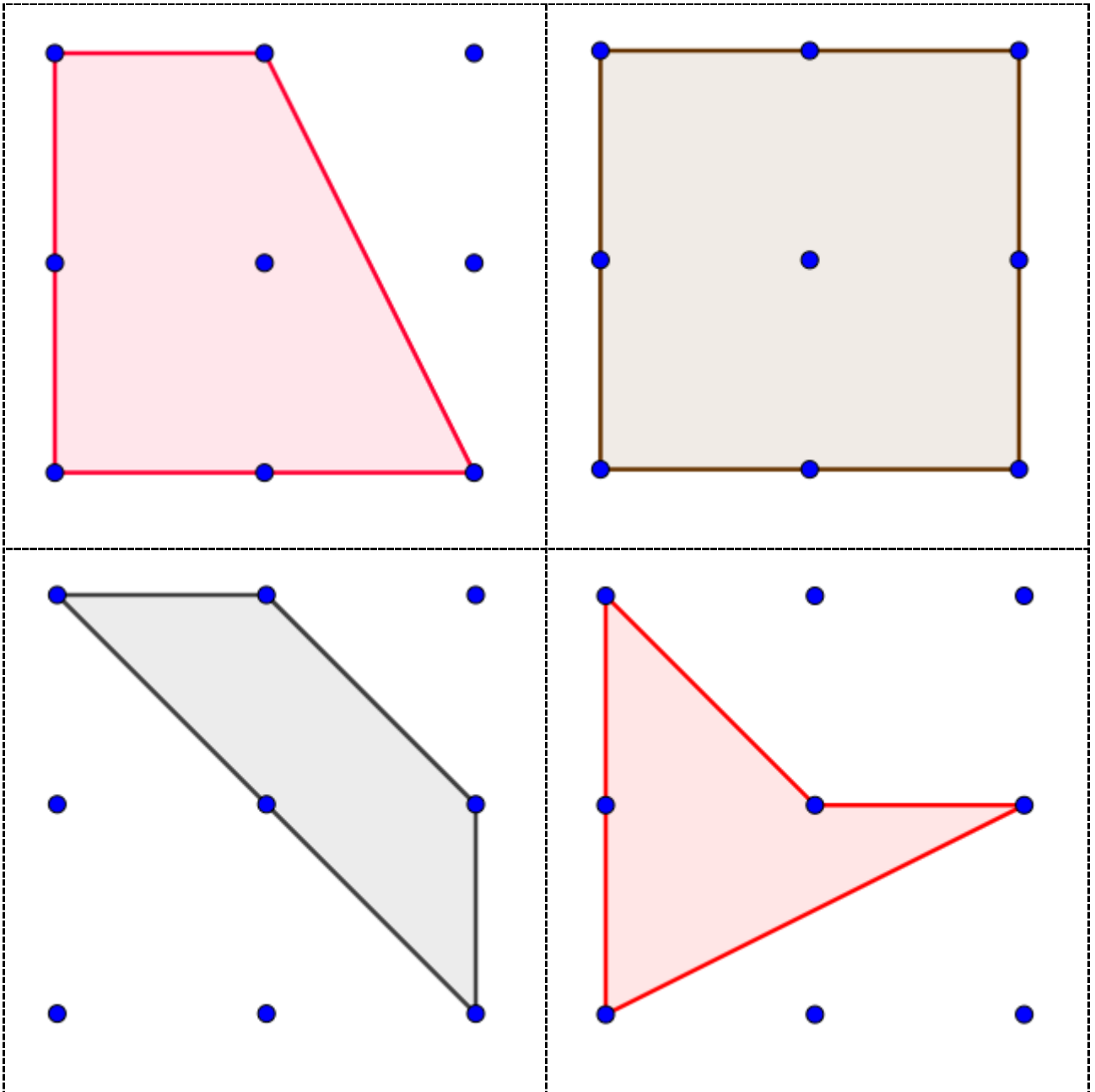
Suggested instructions for this task:

- Each group should be given a set of 9-pin geoboard quadrilateral cards (16 in total) along with text cards (2 cards with suggested text and 2 blank cards provided).
- The group should arrange the cards based on the criteria for classification outlined on the text cards.
- In turn, students in the group should select a card. The student identifies where they believe the card should be placed based on their understanding of the criteria for classification.
- The card can only be placed once the group has reached a consensus.
- The group can postpone the placement of a card only twice during the task and only after a discussion has taken place about its placement. [This task condition is to ensure students engage with some of the more challenging cards. The class teacher is best placed to decide whether this task condition is appropriate]
- The teacher should move around the room gathering evidence about student's knowledge, understanding and learning. Appropriate questioning often provides greater insight.
- Once the task has been completed, a plenary discussion is recommended. This should be guided by the evidence gathered by the teacher during the task. Effective questioning has the potential to enrich the class discussion and student learning.



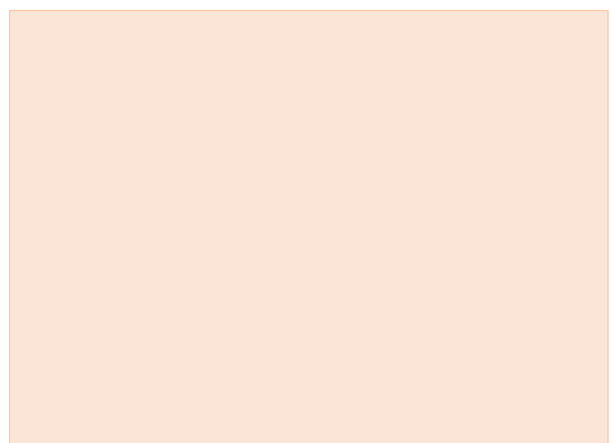
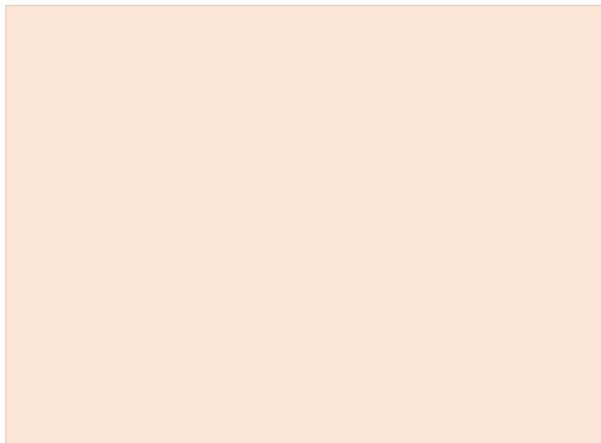




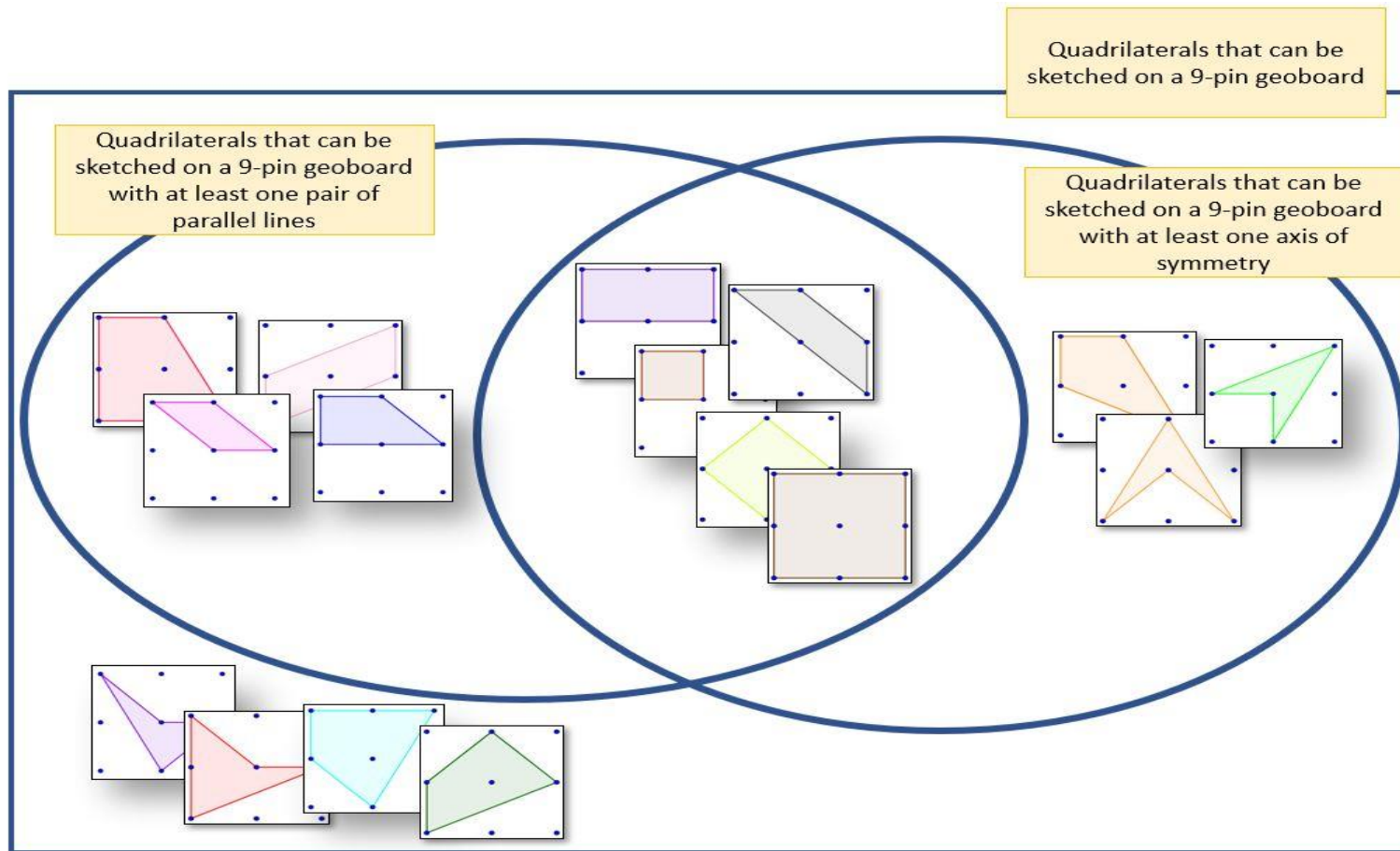


Quadrilaterals that can be sketched on a 9-pin geoboard with at least one pair of parallel lines

Quadrilaterals that can be sketched on a 9-pin geoboard with at least one axis of symmetry



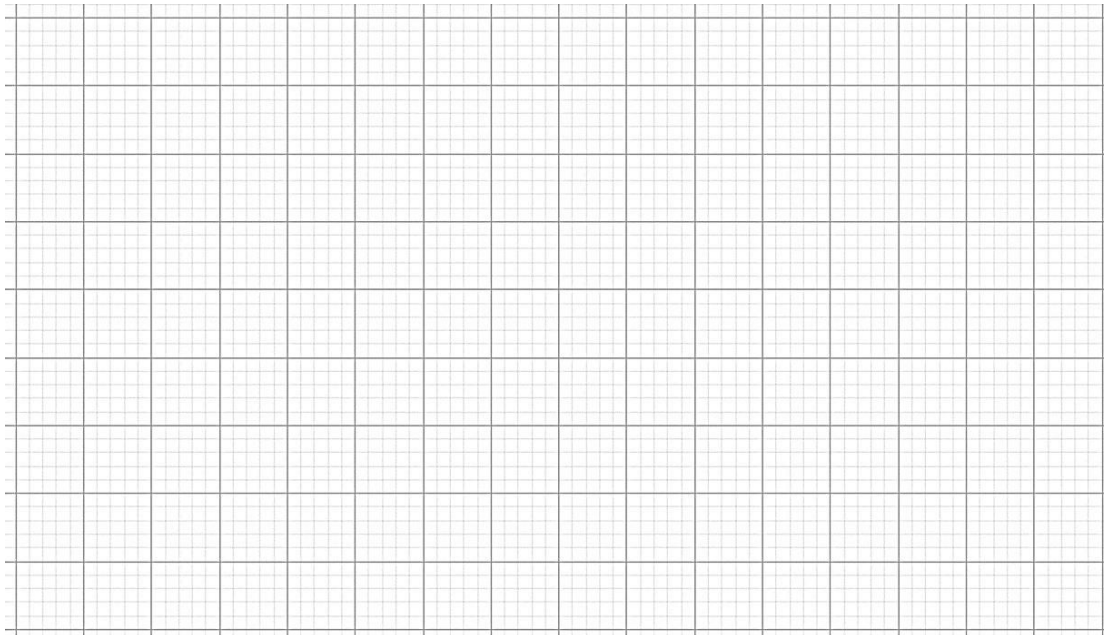
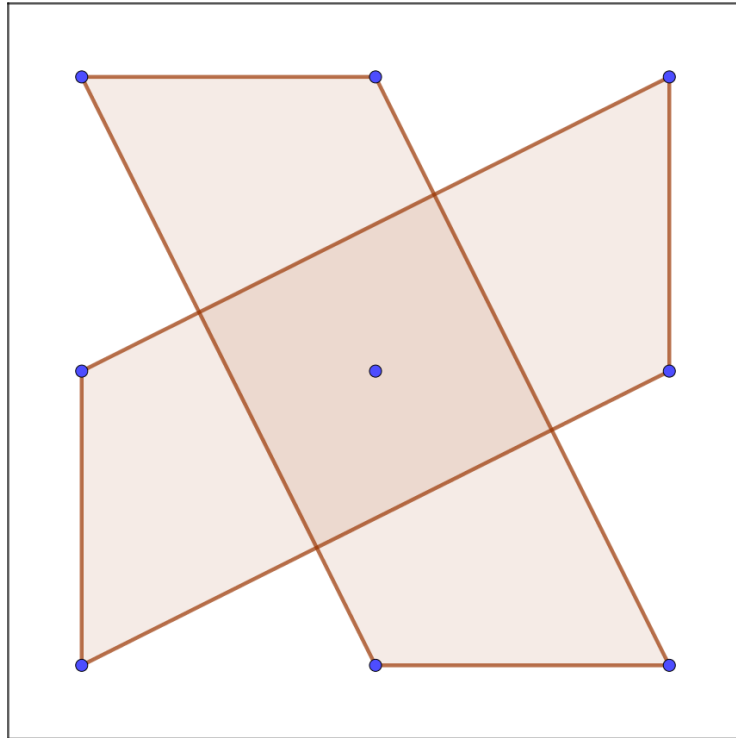
Suggested Solution:



Overlap Task

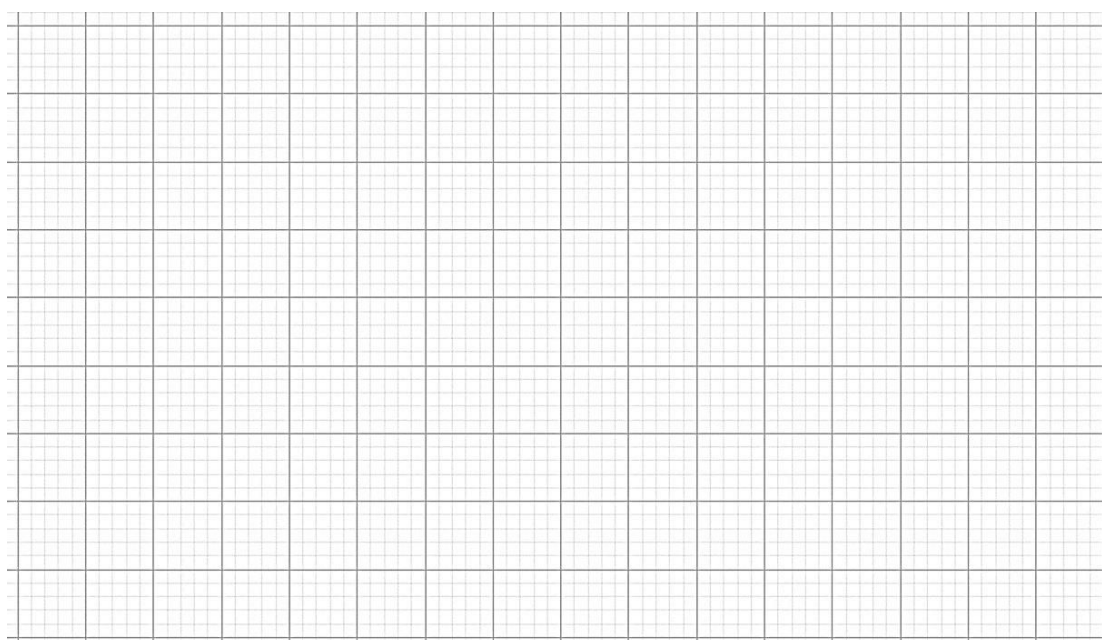
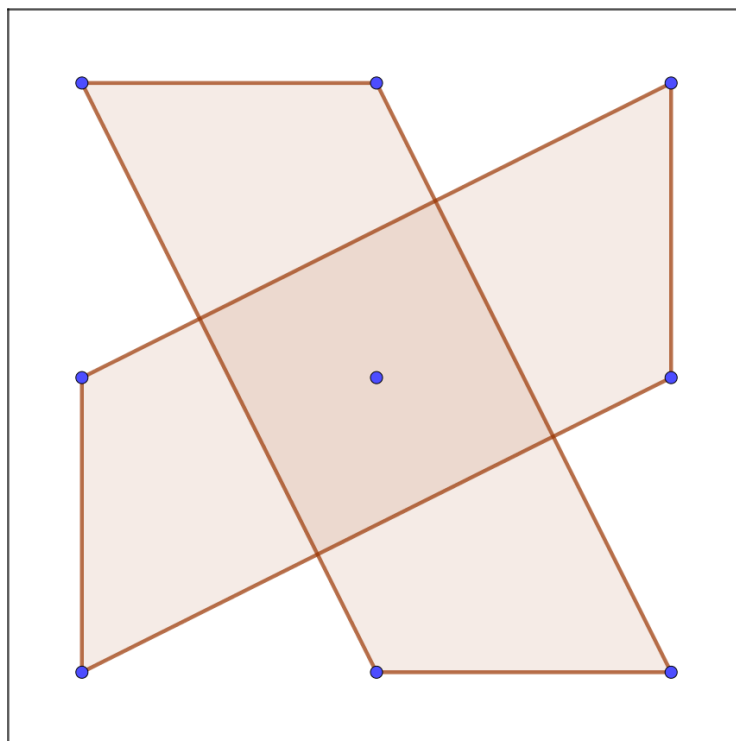
The shape below has been made by overlapping two parallelograms on a 9-pin geoboard.

Consider, with justification, if the shape created by the overlap is a square?



Overlap Task – Variation

The shape below has been made by overlapping two parallelograms on a 9-pin geoboard. Find the area of the overlap.



Deepening the learning - Question Stems

Using question stems is a useful strategy that can be used to probe and ultimately deepen the learning for the student. It is also a formative assessment strategy that should be explicitly modelled for students as a problem-solving skill. It gives students stems for questions that they can ask themselves or their peers when navigating a problem or task.

Why does...?	What would happen if...?	Why do you think...?
How does this relate...?	Does anyone have a different way to explain...?	What facts support...?
Is...always true?	Does...when...?	How would/could you use...?
How could we find out if...?	What could the reason for...be?	What examples show...?
What would happen to the pattern if...?	What other way could you...?	What rule explains...?
Does it keep happening?	What needs to be changed so that...?	Explain your conclusion?
How can we be sure that...?	What do you think the problem is asking?	What can/must be [added/removed/altered] in order to [allow/ensure/contradict...?

Establishing Student Context – Terminology Inventory Probe

A Terminology Inventory Probe (TIP) is a formative assessment strategy that can be used to determine the level of proficiency of students' mathematical language and terminology. It can be adapted and used in the following ways:

- at the beginning, during or end of a unit of learning
- in the preceding lesson(s) before a task-based learning experience to help establish the student readiness
- at the beginning of a task-based learning experience

This editable TIP has been reproduced from:

Keeley, P. and Tobey, C. (2011) *Mathematics formative assessment: 75 Practical strategies for linking assessment, instruction and learning* Corwin Publishing

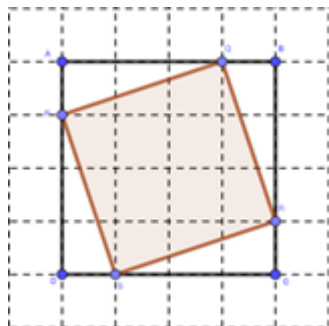
It can be found on the JCT Maths website [here](#).

<u>Terminology Inventory Probe - X</u>		
<p><u>X</u></p> <p><input type="checkbox"/> I have never heard of this.</p> <p><input type="checkbox"/> I have heard of this but I'm not sure what it means.</p> <p><input type="checkbox"/> I have some idea what it means.</p> <p><input type="checkbox"/> I know what it is, and I can explain it.</p>	<p><u>X</u></p> <p><input type="checkbox"/> I have never heard of this.</p> <p><input type="checkbox"/> I have heard of this but I'm not sure what it means.</p> <p><input type="checkbox"/> I have some idea what it means.</p> <p><input type="checkbox"/> I know what it is, and I can explain it.</p>	<p><u>X</u></p> <p><input type="checkbox"/> I have never heard of this.</p> <p><input type="checkbox"/> I have heard of this but I'm not sure what it means.</p> <p><input type="checkbox"/> I have some idea what it means.</p> <p><input type="checkbox"/> I know what it is, and I can explain it.</p>
<p><u>X</u></p> <p><input type="checkbox"/> I have never heard of this.</p> <p><input type="checkbox"/> I have heard of this but I'm not sure what it means.</p> <p><input type="checkbox"/> I have some idea what it means.</p> <p><input type="checkbox"/> I know what it is, and I can explain it.</p>	<p><u>X</u></p> <p><input type="checkbox"/> I have never heard of this.</p> <p><input type="checkbox"/> I have heard of this but I'm not sure what it means.</p> <p><input type="checkbox"/> I have some idea what it means.</p> <p><input type="checkbox"/> I know what it is, and I can explain it.</p>	<p><u>X</u></p> <p><input type="checkbox"/> I have never heard of this.</p> <p><input type="checkbox"/> I have heard of this but I'm not sure what it means.</p> <p><input type="checkbox"/> I have some idea what it means.</p> <p><input type="checkbox"/> I know what it is, and I can explain it.</p>
<p><u>X</u></p> <p><input type="checkbox"/> I have never heard of this.</p> <p><input type="checkbox"/> I have heard of this but I'm not sure what it means.</p> <p><input type="checkbox"/> I have some idea what it means.</p> <p><input type="checkbox"/> I know what it is, and I can explain it.</p>	<p><u>X</u></p> <p><input type="checkbox"/> I have never heard of this.</p> <p><input type="checkbox"/> I have heard of this but I'm not sure what it means.</p> <p><input type="checkbox"/> I have some idea what it means.</p> <p><input type="checkbox"/> I know what it is, and I can explain it.</p>	<p><u>X</u></p> <p><input type="checkbox"/> I have never heard of this.</p> <p><input type="checkbox"/> I have heard of this but I'm not sure what it means.</p> <p><input type="checkbox"/> I have some idea what it means.</p> <p><input type="checkbox"/> I know what it is, and I can explain it.</p>
<p><u>X</u></p> <p><input type="checkbox"/> I have never heard of this.</p> <p><input type="checkbox"/> I have heard of this but I'm not sure what it means.</p> <p><input type="checkbox"/> I have some idea what it means.</p> <p><input type="checkbox"/> I know what it is, and I can explain it.</p>	<p><u>X</u></p> <p><input type="checkbox"/> I have never heard of this.</p> <p><input type="checkbox"/> I have heard of this but I'm not sure what it means.</p> <p><input type="checkbox"/> I have some idea what it means.</p> <p><input type="checkbox"/> I know what it is, and I can explain it.</p>	<p><u>X</u></p> <p><input type="checkbox"/> I have never heard of this.</p> <p><input type="checkbox"/> I have heard of this but I'm not sure what it means.</p> <p><input type="checkbox"/> I have some idea what it means.</p> <p><input type="checkbox"/> I know what it is, and I can explain it.</p>

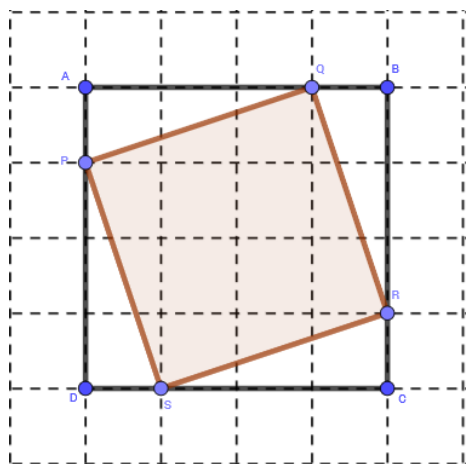
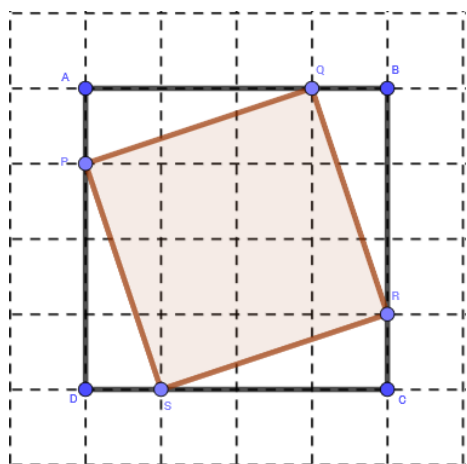
Tasks from CPD days

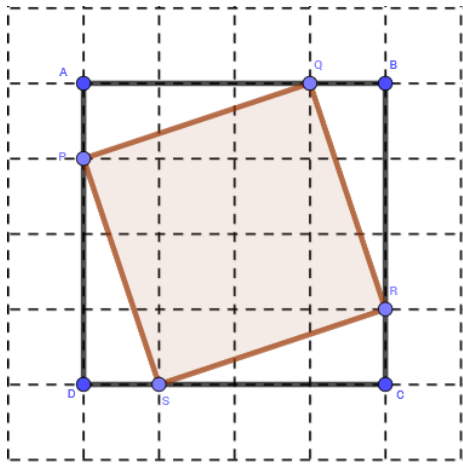
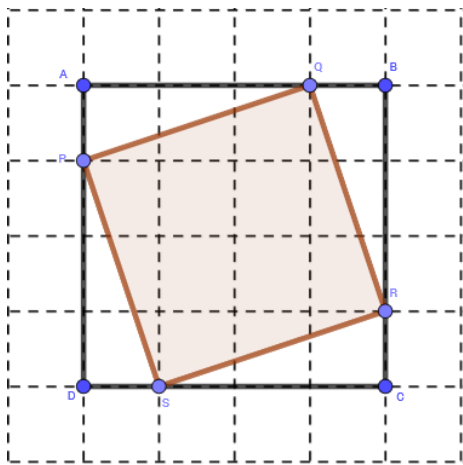
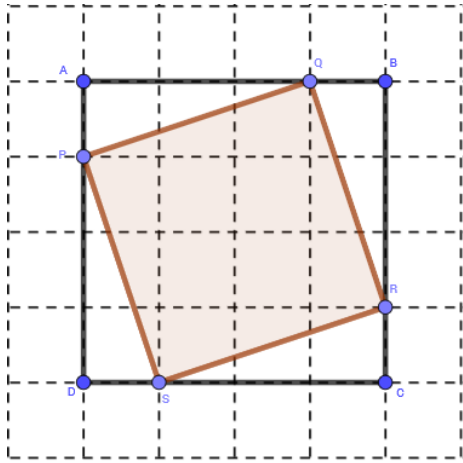
Finding the Area of the Quadrilateral

ABCD is a square. Find the area of the quadrilateral PQRS in as many ways as possible



Sourced from: <http://www.projectmaths.ie/for-teachers/conferences/maths-counts-2015/>

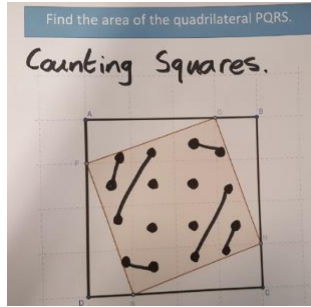




Some Possible Solutions (not exhaustive):

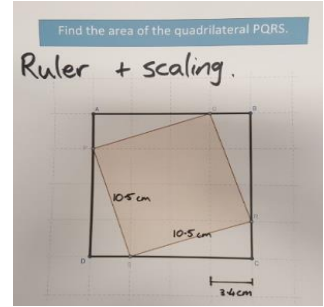
Solution 1

Counting Squares



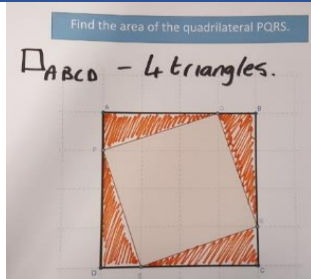
Solution 2

Ruler and Scaling



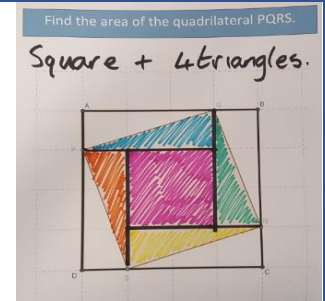
Solution 3

4 Triangles



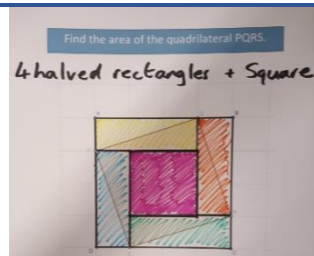
Solution 4

Square and 4 Triangles



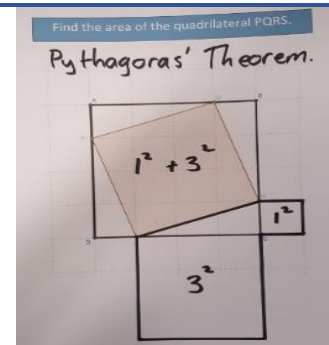
Solution 5

4 halved rectangles and Square



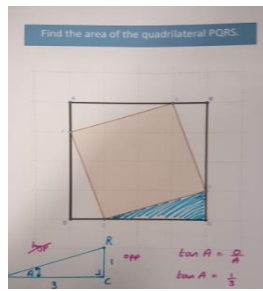
Solution 6

Pythagoras' Theorem



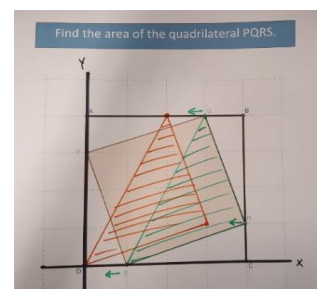
Solution 7

Trigonometry



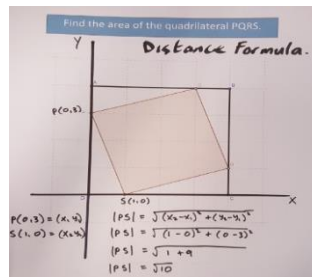
Solution 8

Coordinate
Geometry Area of
Triangle



Solution 9

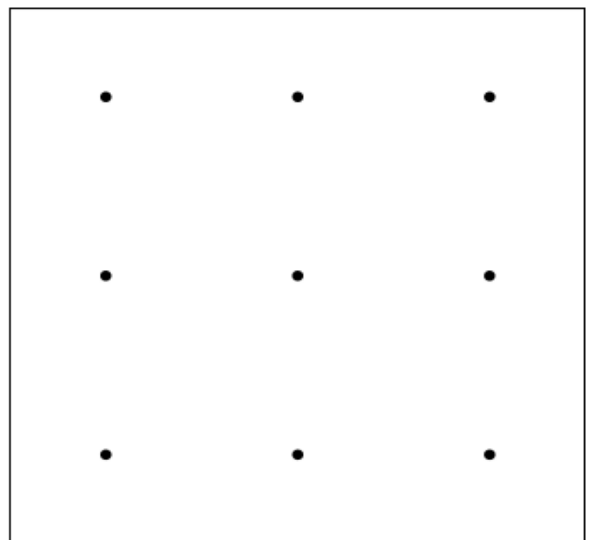
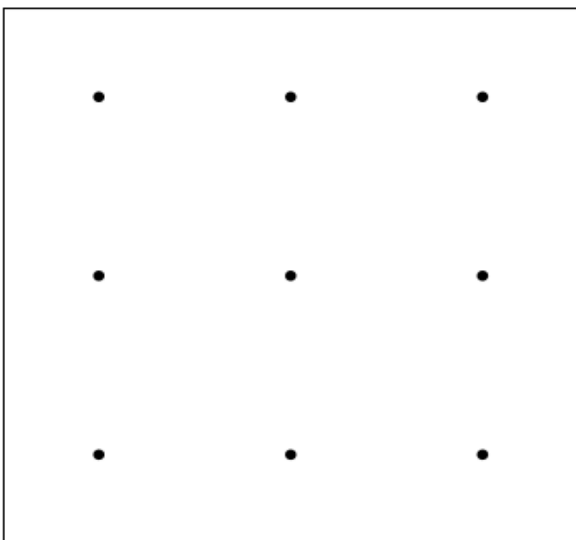
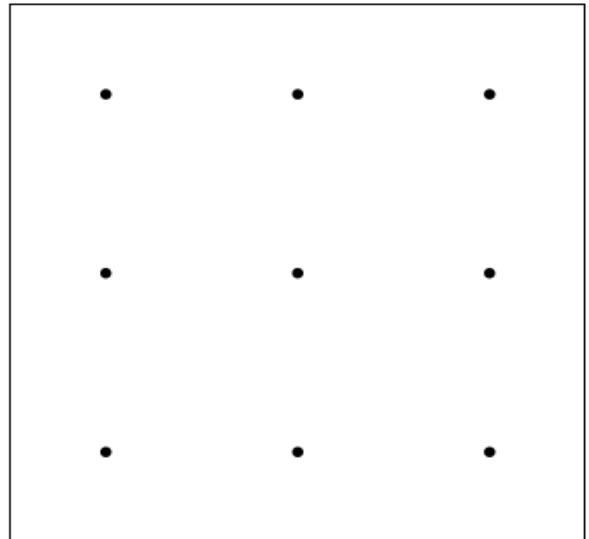
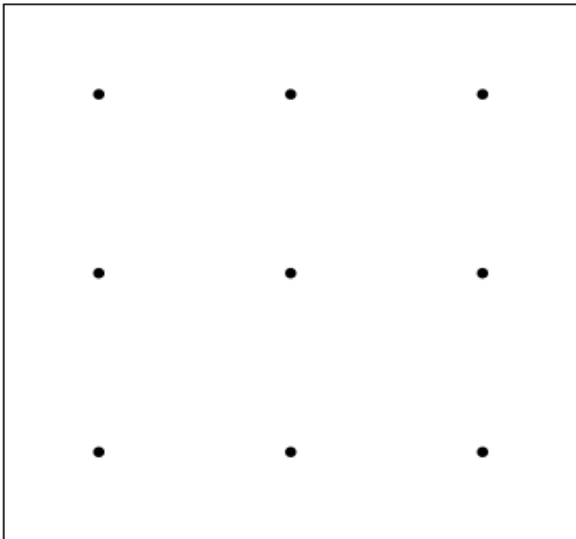
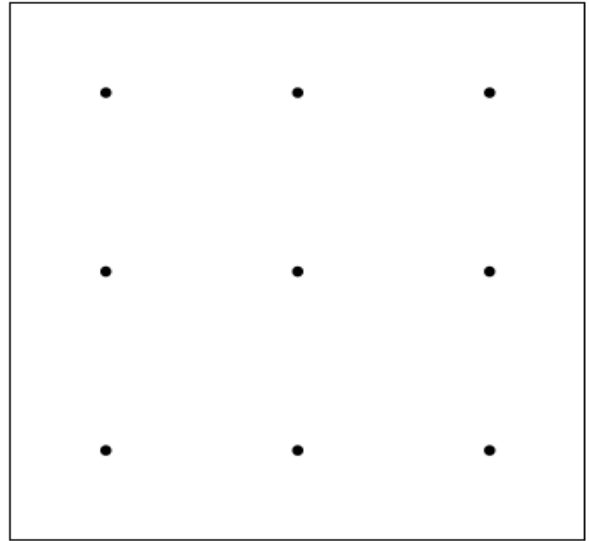
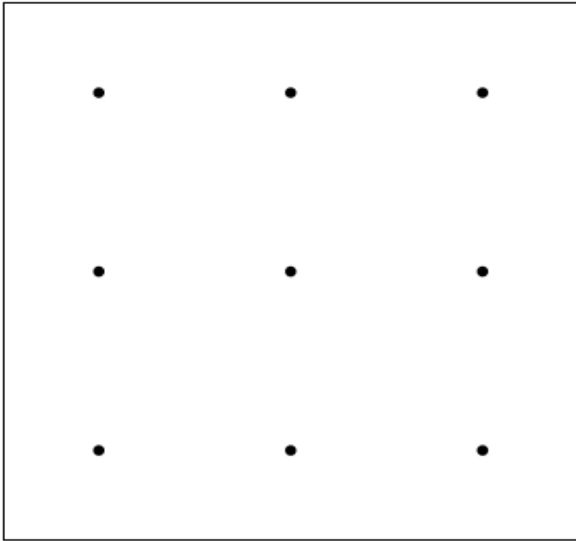
Coordinate
Geometry -
Distance Formula



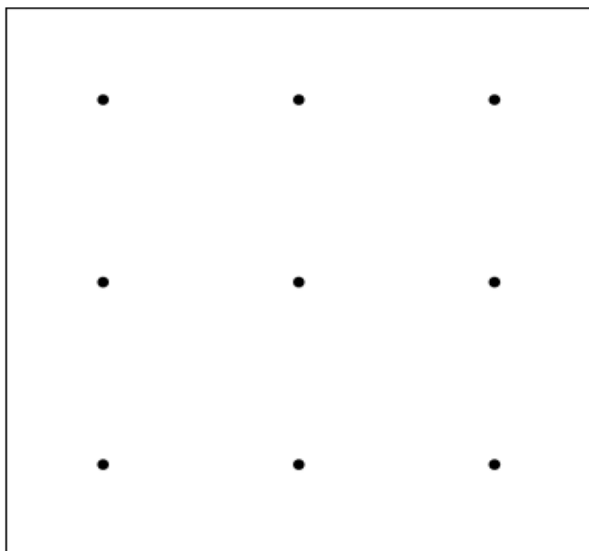
Learning Experience with Geoboards

Sketch as many non-congruent triangles as possible on the following 9-pin geoboards.

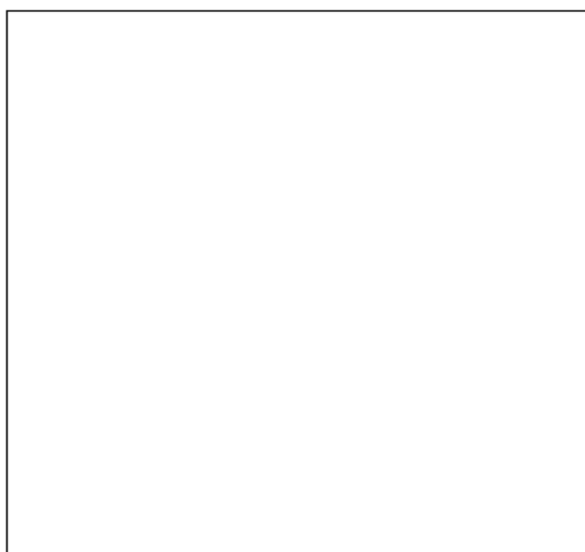
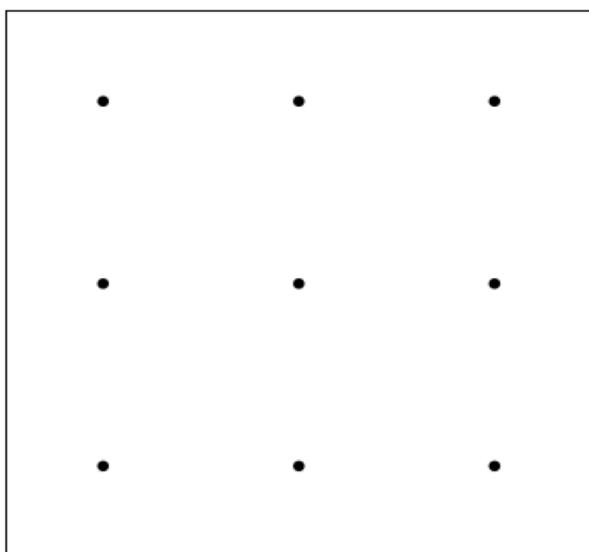
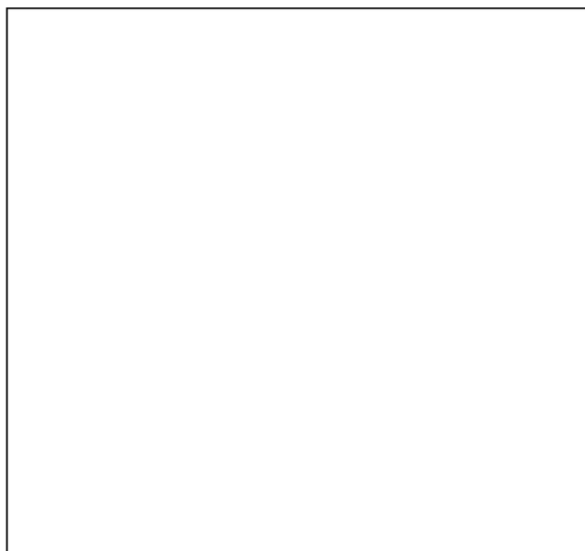
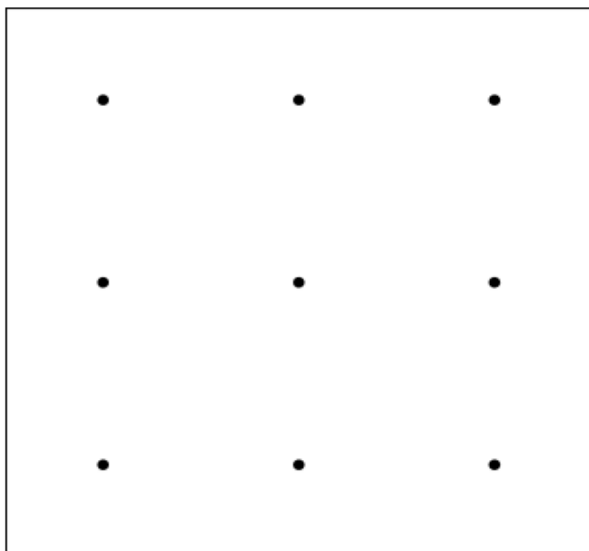
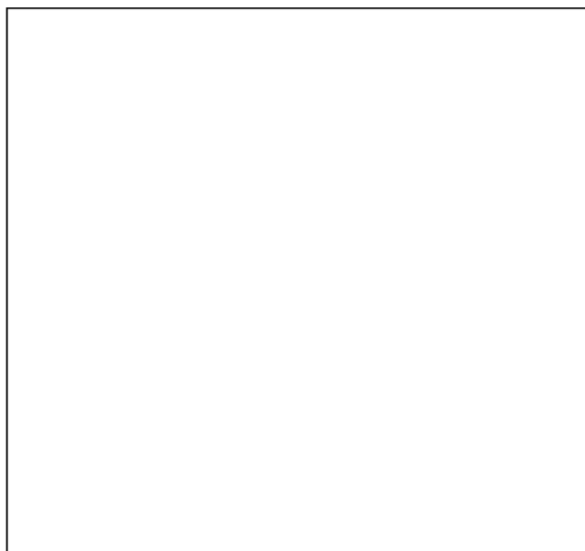
Then order the triangles in terms of their area, from smallest to largest.



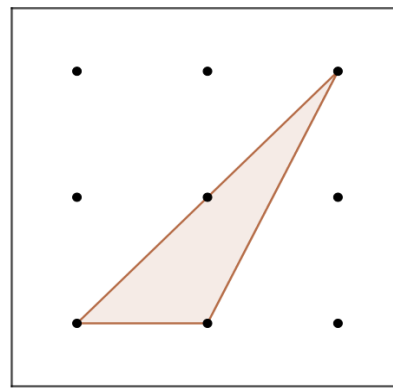
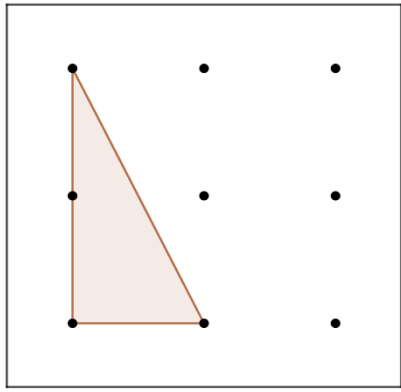
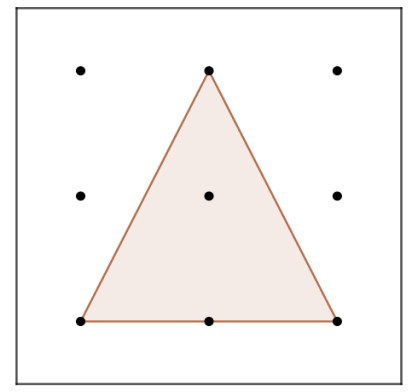
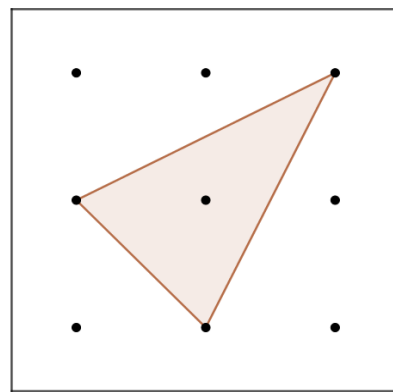
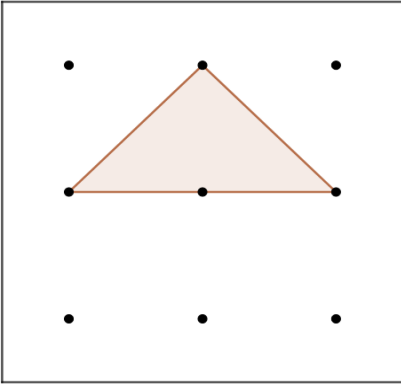
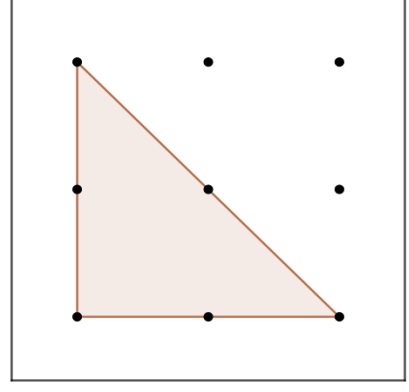
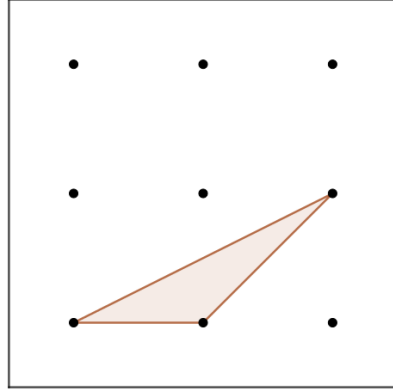
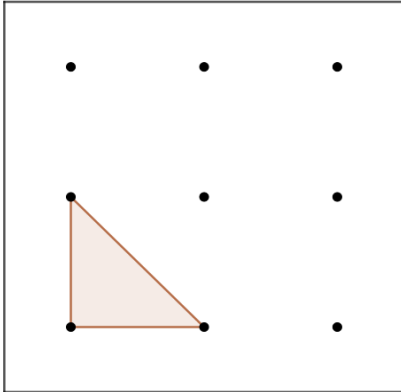
Sketch your triangles here



Write your area calculations



Solutions (excluding congruent triangles):

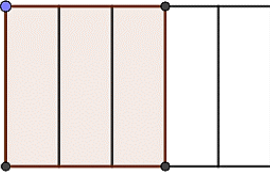
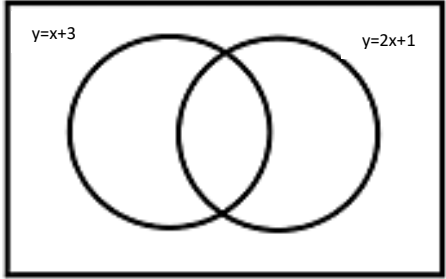


Effective Questioning for Deeper Learning

Open questions promote deeper engagement with learning

Closed questions generally allow for a single response

In the table below, a closed question has been used to generate a possible open question

Closed question	Possible open question
Calculate the perimeter of a rectangle with length 6 units and breadth 5 units?	If a rectangle has a perimeter of 22 units, what are its possible dimensions?
Calculate $\frac{3}{4} + \frac{3}{12}$	Give an example of two fractions that add to 1.
Round 5.7347 correct to one decimal place	A number has been rounded to 5.7 What might the original number have been?
What proportion of the shape below is shaded? 	Shade 3/5 of a rectangle in as many ways as possible
Find the point of intersection of $y=2x+1$ and $y=x+3$	 Place at least one set of coordinates in each part of the Venn Diagram
Calculate the mean of 2, 6, 8 and 4?	Create a list of four numbers whose mean is 5.
Calculate 12×0.5	Give me an example where multiplying two numbers gives an answer that is smaller than either.

Consecutive Numbers Task

Purpose: To develop students' ability to generate and evaluate mathematical arguments

'Natural numbers can be expressed as the sum of two or more consecutive positive whole numbers'

**Investigate this statement.
What conclusions can be drawn?
Justify your reasoning.**

This task can be used to develop students' understanding of connections between the Number and Algebra and Functions strands and can be used to strengthen students' understanding of generating and evaluating mathematical arguments. It is suggested that this should be a focus of the plenary. Please note, this task is not an example of a Classroom-Based Assessment.

Students should be encouraged to explain and justify their reasoning in relation to the generalisation or mathematical argument they generate. Reasoning is often done verbally, and accurate use of mathematical language should be developed. Students should also be encouraged to use the *Problem-Solving Toolkit* from CPD day 2018-19 to identify and note the tools and skills they are using and learning about. Where appropriate, students should be alerted to the difference between justification and formal proof.

The task is aligned to many of the Unifying strand's learning outcomes from the Junior Cycle Mathematics specification. For example, the teacher may choose to focus on:

- U4 represent a mathematical situation in a variety of different ways, including numerically, algebraically, graphically, physically, in words; and to interpret, analyse, and compare such representations
- U10 evaluate different possible solutions to a problem, including evaluating the reasonableness of the solutions, and exploring possible improvements and/or limitations of the solutions (if any)
- U11 generate and evaluate mathematical statements or conjectures based on specific instances

The task is aligned to the following contextual strand learning outcomes from the Junior Cycle Mathematics specification:

- N1 investigate the representation of numbers and arithmetic operations so that they can:
 - a) represent the operations of addition, subtraction, multiplication, and division in \mathbb{N} , \mathbb{Z} , and \mathbb{Q} using models including the number line, decomposition, and accumulating groups of equal size
 - c) explore numbers written as a^b
- N4 analyse numerical patterns in different ways, including making out tables and graphs and continue such patterns
- AF1 investigate patterns and relationships (linear, quadratic, doubling and tripling) in numbers, spatial patterns and real-world phenomena involving change so that they can:
 - a) represent these patterns and relationships in tables and graphs

b) generate a generalised expression for linear **and quadratic** patterns in words and algebraic expressions and fluently convert between each representation

c) categorise patterns as linear, non-linear, **quadratic, and exponential (doubling and tripling)** using their defining characteristics as they appear in the different representations

- AF2 investigate situations in which letters stand for quantities that are variable so that they can:
 - a) generate and interpret expressions in which letters stand for numbers
 - b) find the value of expressions given the value of the variables

It may be necessary to reduce the difficulty level of the task by selectively using effective questioning to guide students towards a generalisation or mathematical argument. It is recommended, however, that scaffolding be kept to a minimum to allow students to apply their knowledge in unfamiliar situations and create a need for the construction of new knowledge. If the task is scaffolded students may engage with it multiple times over the three years of Junior Cycle. Teachers can encourage student dialogue using effective questioning and active listening. Below are some examples of questions that reinforce a student's prior knowledge and/or challenge their misconceptions. These questions may also provide an opportunity for extension and enrichment for students who may have fully grasped the concepts in primary school. A Quick Reference Guide to the Links between the Primary and Post-Primary Curricula can be found [here](#).

This task can be used with students at the beginning, during or at the end of their initial study of Natural Numbers. If the task is used at the beginning or during a unit of learning the task is formative as it provides an opportunity for the class teacher to assess the level of knowledge of their students regarding the identification, sorting, organising and categorisation of various patterns which arise, and adapt the focus of learning.

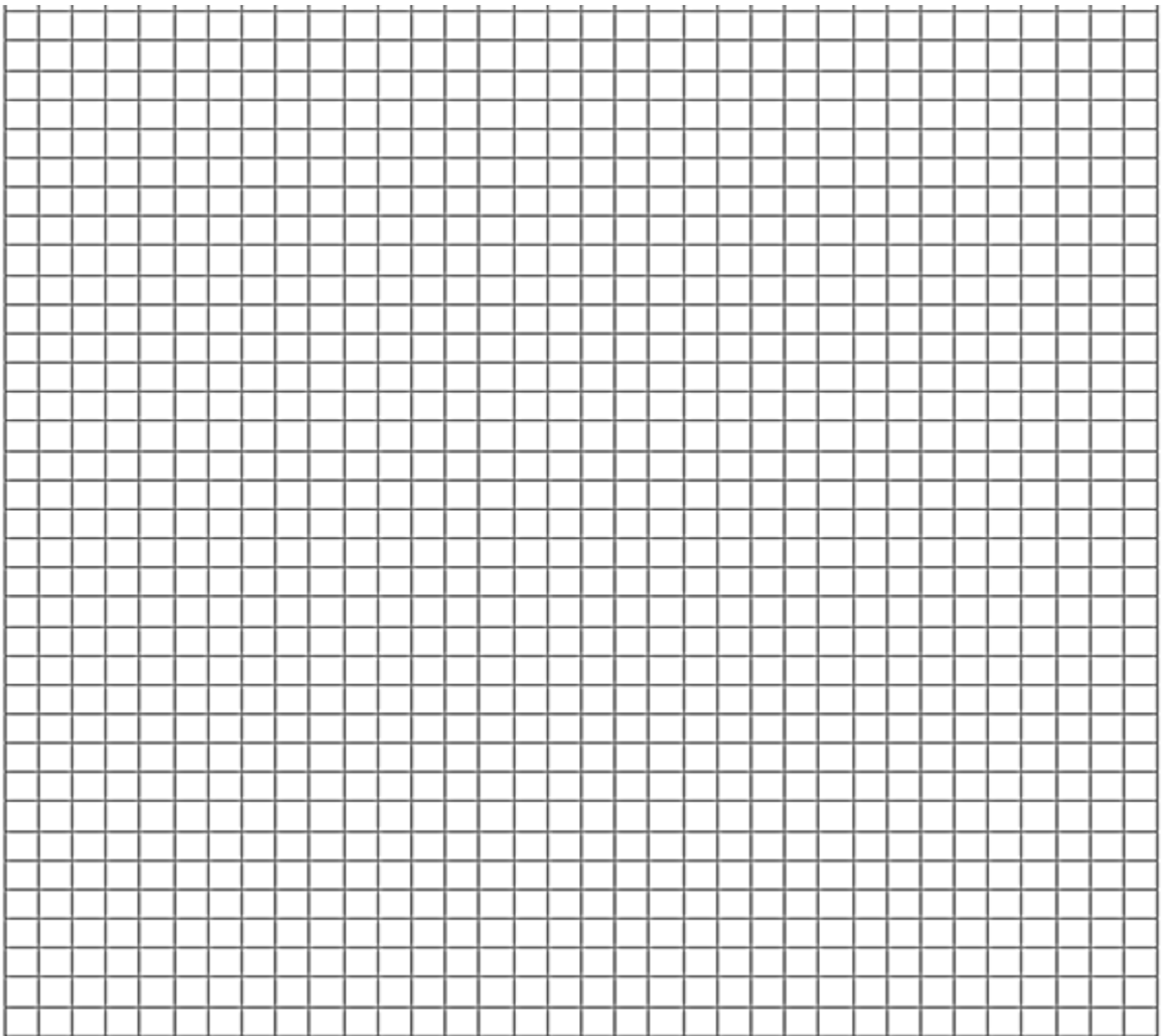
To promote students' mathematical thinking and discussion, and to generate rich classroom dialogue it is recommended that the task be undertaken in groups of between two and four students.

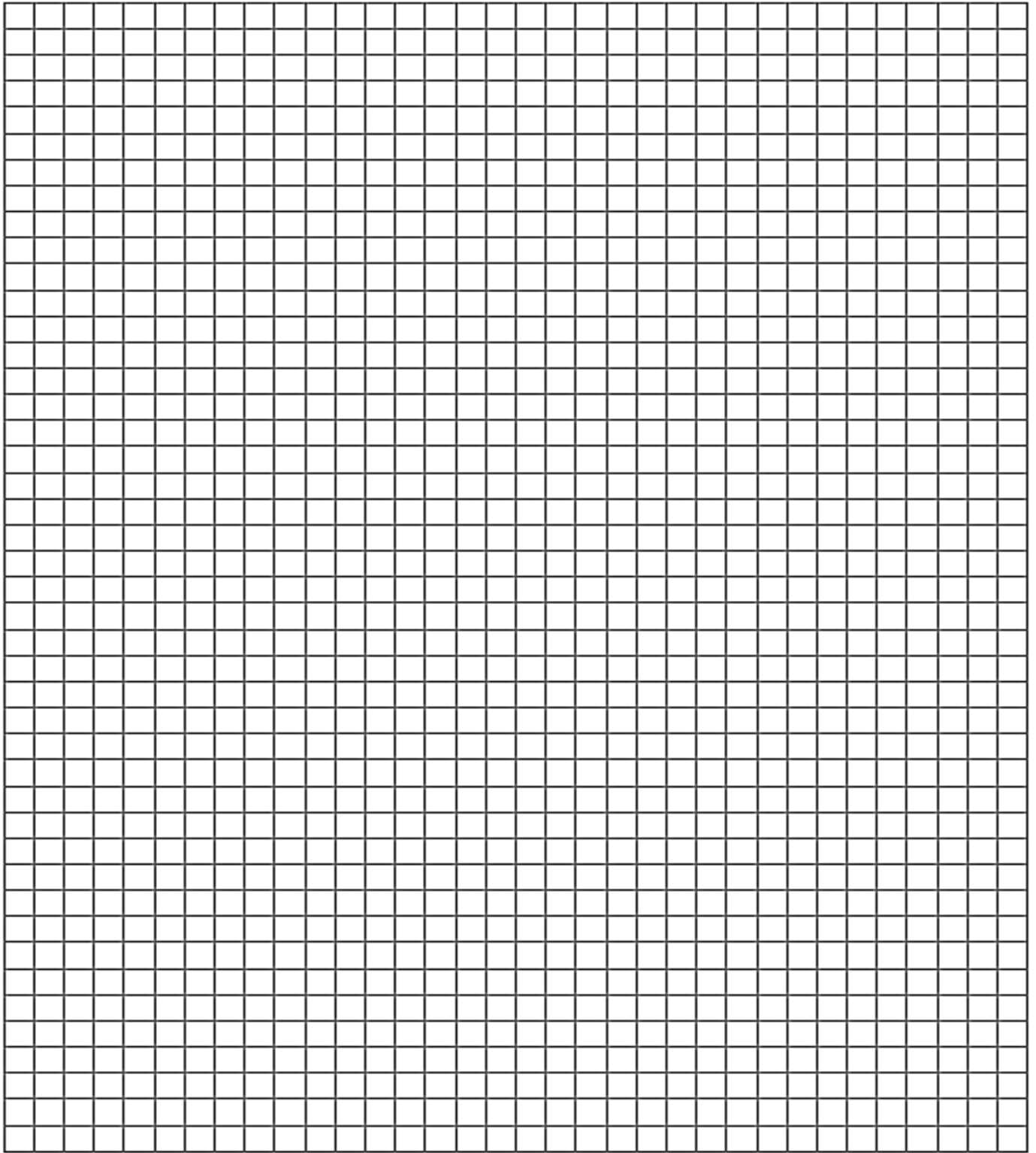
Suggested instructions for using this task:

- The teacher should introduce the task by formatively assessing the students current understanding of *Natural Numbers, Consecutive Numbers, Positive Numbers and Whole Numbers*. (It may be useful to check Learner Context using a Terminology Inventory Probe [here](#))
- Each group should be given copy of the Consecutive Numbers Task.
- An initial opportunity to engage with the statement should be limited to *5 minutes*. Students should be encouraged to use examples to support their decisions.
- Teachers can encourage classroom dialogue through effective questioning and active listening.
- After consensus has been reached on the statement, the groups should be given a second opportunity to engage with the statement and encouraged to use the *Problem-Solving Toolkit* to identify the mathematical tools they have used or learned about.
- The teacher should move around the room informally gathering information (evidence) about student's knowledge and learning. Appropriate questioning often provides greater insight.
- Once the task has been completed, a plenary discussion is recommended. This should be informed by an information (evidence) gathering process and where appropriate questioning has the potential to enrich the class discussion and student learning.

‘Natural numbers can be expressed as the sum of two or more consecutive positive whole numbers’

**Investigate this statement.
What conclusions can be drawn?
Justify your reasoning.**

A large grid of graph paper, consisting of 20 columns and 30 rows of small squares, intended for students to work out their investigation and justify their reasoning.



Sample Questions for Deeper Learning (This is not an exhaustive list)

<p style="text-align: center;">'Natural numbers can be expressed as the sum of two or more consecutive positive whole numbers'</p> <p style="text-align: center;">Investigate this statement. What conclusions can be drawn? Justify your reasoning.</p>	<p>What examples show that this is <i>Always</i>, <i>Sometimes</i>, <i>Never</i> true?</p> <p>Can you explain your conclusion?</p> <p>What would happen if you examined odd numbers only?</p> <p>What would happen if you examined the even numbers only?</p>
<p style="text-align: center;">$1 + 2 = 3$ $2 + 3 = 5$ $3 + 4 = 7$ $4 + 5 = 9 \dots$</p>	<p>What happens if you sum two consecutive numbers?</p> <p>Does this keep happening?</p> <p>What examples show this?</p> <p>What rule explains this best?</p> <p>Can you make a generalisation?</p>
<p style="text-align: center;">$1 + 2 + 3 = 6$ $2 + 3 + 4 = 9$ $3 + 4 + 5 = 12$ $4 + 5 + 6 = 15 \dots$</p>	<p>What happens if you sum three consecutive numbers?</p> <p>Does this keep happening?</p> <p>What examples show this?</p> <p>What rule explains this best?</p> <p>Can you make a generalisation?</p>
<p style="text-align: center;">$2 =$ $6 = 1+2+3$ $3 = 1 + 2$ $7 = 3+4$ $4 =$ $8 =$ $5 = 2 + 3$ $9 = 2+3+4$</p>	<p>What would happen if you examined the numbers that cannot be made by summing consecutive numbers?</p> <p>Which natural numbers cannot be represented as a sum of consecutive numbers?</p> <p>Is there a pattern here?</p> <p>Can you make a generalisation?</p>

Possible Extension Tasks

Below are some alternative extension tasks that allow for the links between Number and Algebra to be exposed.

- The difference of two consecutive numbers is an odd number.
- The sum of two consecutive triangular numbers is a square number.
- The sum of three consecutive triangular numbers is one more than three times the middle triangular number.
- The sum of three consecutive numbers is divisible by three.
- The sum of five consecutive numbers is divisible by five.
- The sum of four consecutive numbers is divisible by two.
- The sum of six consecutive numbers is divisible by three.
- One more than the sum of four consecutive numbers is even.
- The product of two consecutive numbers is divisible by two.
- The product of three consecutive numbers is divisible by six.
- One more than four times the product of two consecutive numbers is a perfect square.
- One more than the product of two numbers differing by two is a perfect square.
- One more than the product of four consecutive numbers is the square of one less than the product of the two middle numbers.
- The product of three consecutive numbers is the middle number less than a perfect cube.
- One more than eight times a triangular number is always a perfect square.

Sourced from: Mason, J. Graham, A., & Johnston-Wilder, S. (2005) *Developing Thinking in Algebra* Paul Chapman Publishing

**Terminology Inventory Probe -
Numbers**

<p><u>Natural numbers</u></p> <ul style="list-style-type: none"> <input type="checkbox"/> I have never heard of these. <input type="checkbox"/> I have heard of these but I'm not sure what they are. <input type="checkbox"/> I have some idea what these are. <input type="checkbox"/> I know what these are and I can give an explanation. 	<p><u>Integers</u></p> <ul style="list-style-type: none"> <input type="checkbox"/> I have never heard of these. <input type="checkbox"/> I have heard of these but I'm not sure what they are. <input type="checkbox"/> I have some idea what these are. <input type="checkbox"/> I know what these are and I can give an explanation. 	<p><u>Real numbers</u></p> <ul style="list-style-type: none"> <input type="checkbox"/> I have never heard of these. <input type="checkbox"/> I have heard of these but I'm not sure what they are. <input type="checkbox"/> I have some idea what these are. <input type="checkbox"/> I know what these are and I can give an explanation.
<p><u>Decimals</u></p> <ul style="list-style-type: none"> <input type="checkbox"/> I have never heard of these. <input type="checkbox"/> I have heard of these but I'm not sure what they are. <input type="checkbox"/> I have some idea what these are. <input type="checkbox"/> I know what these are and I can give an explanation. 	<p><u>Fractions</u></p> <ul style="list-style-type: none"> <input type="checkbox"/> I have never heard of these. <input type="checkbox"/> I have heard of these but I'm not sure what they are. <input type="checkbox"/> I have some idea what these are. <input type="checkbox"/> I know what these are and I can give an explanation. 	<p><u>Whole numbers</u></p> <ul style="list-style-type: none"> <input type="checkbox"/> I have never heard of these. <input type="checkbox"/> I have heard of these but I'm not sure what they are. <input type="checkbox"/> I have some idea what these are. <input type="checkbox"/> I know what these are and I can give an explanation.
<p><u>Consecutive numbers</u></p> <ul style="list-style-type: none"> <input type="checkbox"/> I have never heard of these. <input type="checkbox"/> I have heard of these but I'm not sure what they are. <input type="checkbox"/> I have some idea what these are. <input type="checkbox"/> I know what these are and I can give an explanation. 	<p><u>Odd numbers</u></p> <ul style="list-style-type: none"> <input type="checkbox"/> I have never heard of these. <input type="checkbox"/> I have heard of these but I'm not sure what they are. <input type="checkbox"/> I have some idea what these are. <input type="checkbox"/> I know what these are and I can give an explanation. 	<p><u>Even numbers</u></p> <ul style="list-style-type: none"> <input type="checkbox"/> I have never heard of these. <input type="checkbox"/> I have heard of these but I'm not sure what they are. <input type="checkbox"/> I have some idea what these are. <input type="checkbox"/> I know what these are and I can give an explanation.
<p><u>Square numbers</u></p> <ul style="list-style-type: none"> <input type="checkbox"/> I have never heard of these. <input type="checkbox"/> I have heard of these but I'm not sure what they are. <input type="checkbox"/> I have some idea what these are. <input type="checkbox"/> I know what these are and I can give an explanation. 	<p><u>Sum</u></p> <ul style="list-style-type: none"> <input type="checkbox"/> I have never heard of these. <input type="checkbox"/> I have heard of these but I'm not sure what they are. <input type="checkbox"/> I have some idea what these are. <input type="checkbox"/> I know what these are and I can give an explanation. 	<p><u>Product</u></p> <ul style="list-style-type: none"> <input type="checkbox"/> I have never heard of these. <input type="checkbox"/> I have heard of these but I'm not sure what they are. <input type="checkbox"/> I have some idea what these are. <input type="checkbox"/> I know what these are and I can give an explanation.

An editable version of this T.I.P. can be found [here](#).

Post Box Task


Purpose: To develop students' ability to Question Pose



The purpose of this task is to develop students' ability to question pose. Providing opportunities for students to pose questions that merit mathematical investigation and scrutiny is fundamental to the development of students' abilities to reason, problem-solve, and think critically. Please note, this task is not an example of a Classroom-Based Assessment.

The Assessment Guidelines for Mathematics note that 'defining the problem statement' is an integral aspect of the Mathematical Investigation. The process begins with the student posing and refining the problem to be explored. Students must then identify relevant information that is needed to obtain a solution and/or solution method to the problem posed.

Questions posed can be motivated by curricular content or real-world phenomena. A useful strategy for developing the skill of problem posing in students is the use and integration of multi-modal stimuli such as images, photographs, diagrams, digital manipulatives, video and/or audio recordings. Problem posing is a creative act and is a skill that takes time to develop. Teachers should provide regular opportunities for students to develop these competences.

	Sample Questions (Not an exhaustive list)
	<p>Which post box could hold the most letters? What assumptions have been made?</p> <p>Why is the height of two post boxes different?</p> <p>How often does the postman visit the boxes and what happens with the letters thereafter?</p> <ul style="list-style-type: none">• Efficient use of resources• Sorting office

Look at this image and consider the questions that occur to you.



A large, empty rectangular box with a thin black border, intended for a student's response to the prompt above.

Rectangle Task

Purpose: To develop students' ability to reason mathematically

Two rectangles have the same perimeter but different areas. The area of one is smaller than the area of the other.

Explore and discuss.

The purpose of this task is to develop student understanding of the relationship between perimeter and area and the evaluation of mathematical arguments. Students should be encouraged to explain and justify their reasoning in relation to this task. Reasoning may be done verbally, but the accurate use of mathematical language should be developed. Students should also be encouraged to use the *Problem-Solving Toolkit* from CPD day 2018-19 to identify the tools that they are using when engaging with this task. Please note, this task is not an example of a Classroom-Based Assessment.

This task can be used with students at the beginning, during or at the end of their initial study of Geometry and/or Number. The task allows for the reinforcement of students' knowledge and understanding of area, perimeter and number from primary school. The task also provides an opportunity to extend and enrich the learning for students who have a good knowledge of the basic concepts of area and perimeter. (A Quick Reference Guide to the Links between the Primary and Post-Primary Curricula can be found [here](#).)

The task is aligned to many of the Unifying strand's learning outcomes from the Junior Cycle Mathematics specification. For example, the teacher may choose to focus on:

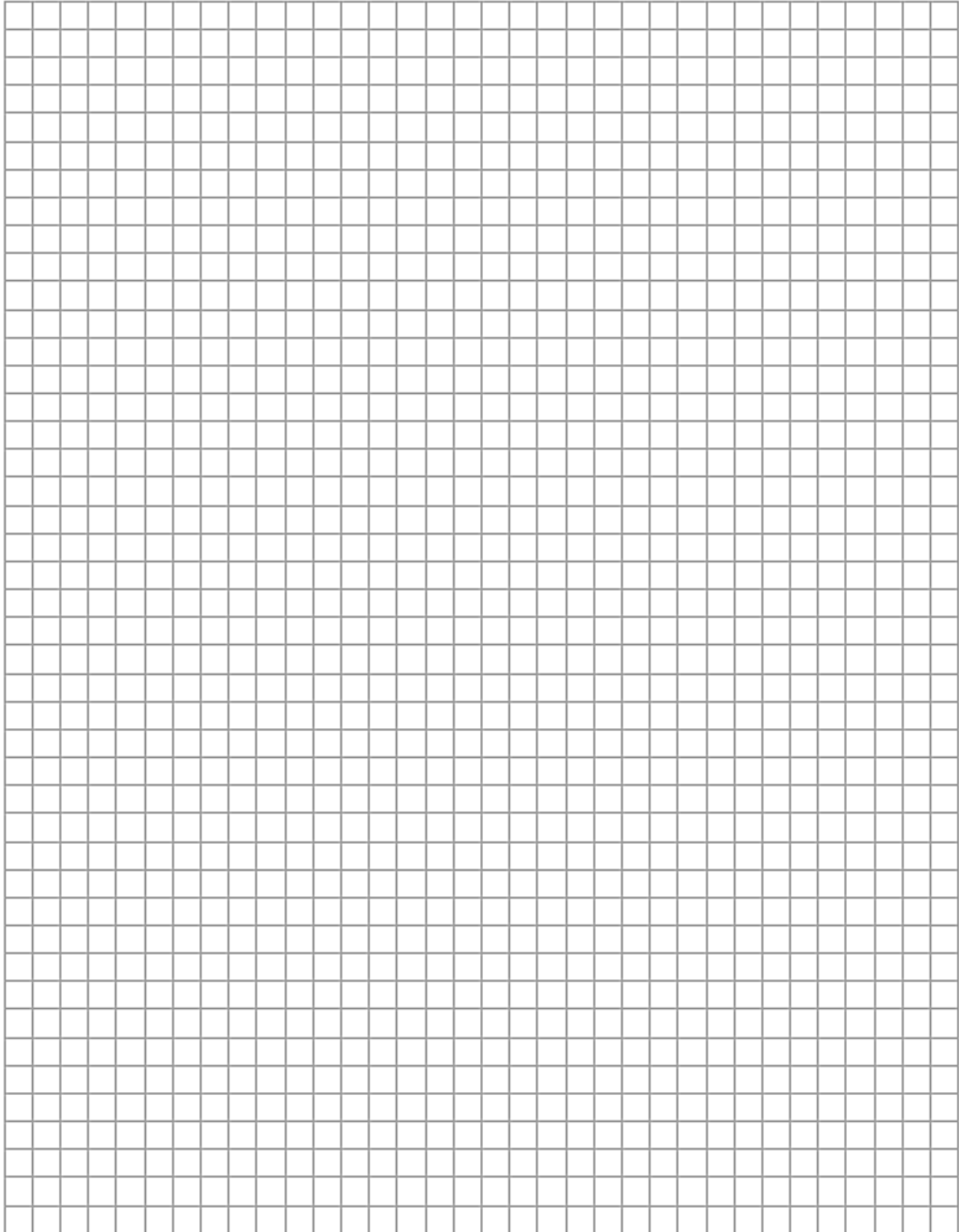
- U4 represent a mathematical situation in a variety of different ways, including: numerically, algebraically, graphically, physically, in words; and to interpret, analyse, and compare such representations
- U7 make sense of a given problem, and if necessary, mathematise a situation
- U8 apply their knowledge and skills to solve a problem, including decomposing it into manageable parts and/or simplifying it using appropriate assumptions
- U12 generate and evaluate mathematical arguments

The task is linked to the following contextual strand learning outcomes from the Junior Cycle Mathematics specification:

- N1 investigate the representation of numbers and arithmetic operations
- GT2 investigate 2-D shapes
- GT3 investigate the concept of proof through their engagement with geometry
- AF2 investigate situations in which letters stand for quantities that are variable

Two rectangles have the same perimeter but different areas. The area of one is smaller than the area of the other.

Explore and discuss.



Possible questioning

Two rectangles have the same perimeter but different areas. The area of one is smaller than the area of the other.

Explore and discuss.

What are your initial thoughts?

Can you find examples where this is true?

From your sketches/tables/diagrams what have you observed?

Can you explain your answer?

How did you find that out?

Why do you think that?

What made you decide to do it that way?

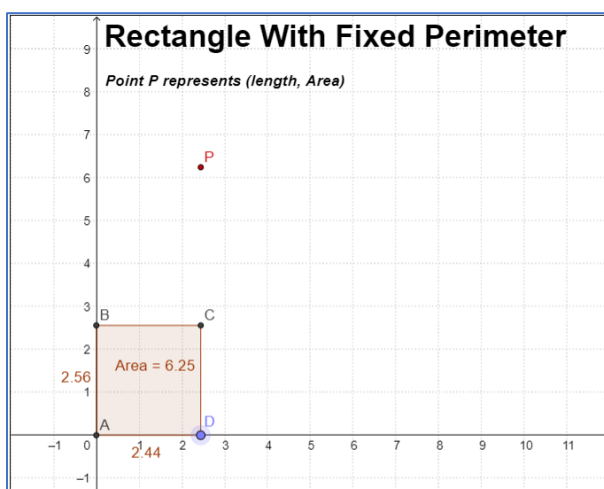
If two rectangles have different dimensions (lengths and widths), could their areas to be the same?

What happens to the area when the difference between the width and length reduces?
Does the area get bigger or smaller?

Possible scaffold

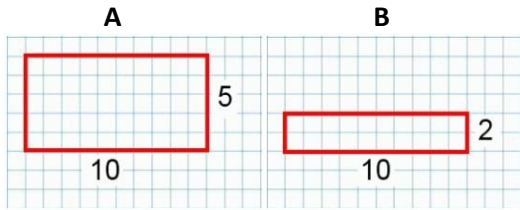
Perimeter	Length	Width	Area
10 units			
10 units			
10 units			
10 units			
10 units			

Find the whole number length and whole number width of every rectangle with a perimeter of 10 units. Record the length, width, and area of each rectangle in the table.



<https://www.geogebra.org/m/kPWP3mDB>

Possible extension questions



www.nrich.org

The rectangle A has a perimeter of 30 units and an area of 50 square units.

The rectangle B has a perimeter of 24 units and an area of 20 square units.

Find the dimension(s) of a rectangle, with a side of length 10 units, whose perimeter and area have the same numerical value

Possible discussion activity

Can be solved algebraically:

$$\text{Perimeter: } 2l + 2w$$

$$\text{Area: } l \times w$$

Equating $2l + 2w = lw$

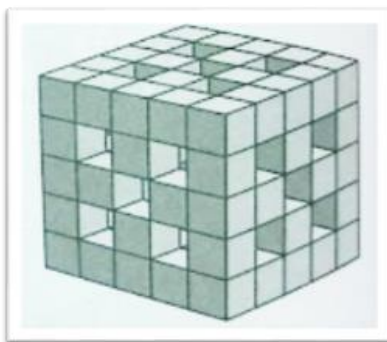
and rearranging for either l or w

$$l = \frac{2w}{w - 2}$$

$$w > 2$$

Cube Task

Purpose: To develop students' ability to Question Pose



The purpose of this task is to strengthen students' ability to pose questions and define a problem in mathematics. Providing opportunities for students to pose questions that merit mathematical investigation and scrutiny is fundamental to the development of students' abilities to reason, problem-solve, and think critically. Please note, this task is not an example of a Classroom-Based Assessment.

Questions posed can be motivated by curricular content or real-world phenomena. A useful strategy for developing the skill of problem posing in students is the use and integration of multi-modal stimuli such as images, photographs, diagrams, digital manipulatives, video and/or audio recordings. The Assessment Guidelines for Mathematics note that 'defining the problem statement' is an integral aspect of the Mathematical Investigation and begins with the student posing and refining the problem to be explored. Problem posing is a creative act and is a skill that takes time to develop. Teachers must provide regular opportunities for students to develop these competences. It is suggested that this should be a focus of the plenary.

This task can be used with students during or at the end of their initial study of Geometry, Number and Algebra. If the task is used during a unit of learning the task is formative, as it provides an opportunity for the class teacher to assess student ability to sketch and analyse geometrical shapes, apply their knowledge of Geometry, Number and Algebra to a problem and justify a conclusion in terms of the original question posed.

The task is aligned to many of the Unifying strand's learning outcomes from the Junior Cycle Mathematics specification. For example, the teacher may choose to focus on:

- U4 Represent a mathematical situation in a variety of different ways, including: numerically, algebraically, graphically, physically, in words; and to interpret, analyse, and compare such representations
- U8 Apply their knowledge and skills to solve a problem, including decomposing it into manageable parts and/or simplifying it using appropriate assumptions

The task is linked to the following contextual strand learning outcomes from the Junior Cycle Mathematics specification:

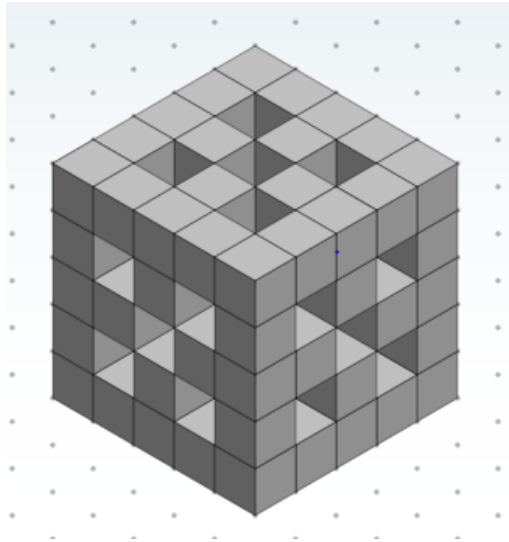
- N4 Analyse numerical patterns in different ways, including making out tables and graphs, and continue such patterns
- GT2 Investigate 3D solids
- AF1 Investigate patterns and relationships (linear, quadratic, doubling and tripling) in number, spatial patterns and real-world phenomena involving change

Useful Digital Resources

The following links are online resources which may be useful tools for students when they are engaging with this task.

<https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Isometric-Drawing-Tool/>

<http://voxelbuilder.com/edit.html#C/2ecc713498db34495ee67e22ecf0f1:A/>



Possible prompts for use with students:

What do you want to know?

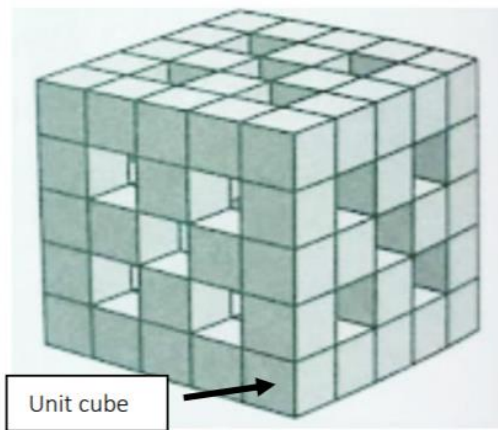
What do you already know?

Are there any concrete materials that could help you?

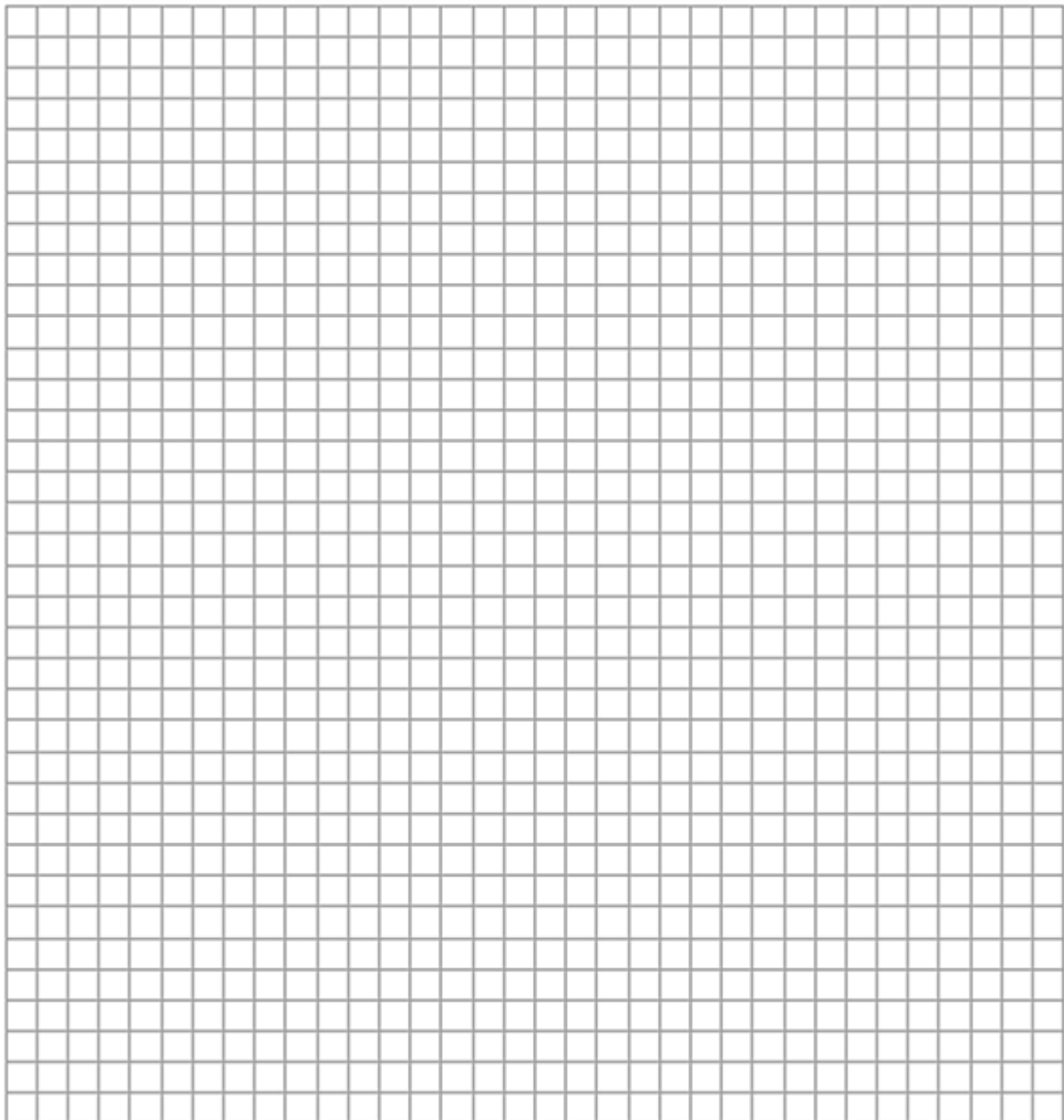
- Building cubes? (Interactive online resources?)
- Isometric graph paper for diagrams?

What assumptions are you making?

- Is the core solid?
- Is the pattern maintained on the sides that are not visible?



Look at this image and consider the questions you may pose based on it.



Cutting Shapes Task

Purpose: To develop students' ability to generate and evaluate mathematical arguments

'If a piece is cut off a two-dimensional shape it reduces both the area and the perimeter.'

Investigate this statement.
What conclusions can be drawn?
Justify your reasoning.

This task can be used to develop students' understanding of the generation and evaluation of mathematical arguments. It is suggested that this should be a focus of the plenary. Please note, this task is not an example of a Classroom-Based Assessment.

Students should be encouraged to explain and justify their reasoning in relation to the generalisation or mathematical argument they generate. Reasoning is often done verbally, and accurate use of mathematical language should be developed. Students should also be encouraged to use the *Problem-Solving Toolkit* from CPD day 2018-19 to identify and note the tools and skills they are using and learning about. Where appropriate, students should be alerted to the difference between reasoned justification and formal proof.

The task is aligned to many of the Unifying strand's learning outcomes from the Junior Cycle Mathematics specification. For example, the teacher may choose to focus on:

- U4 Represent a mathematical situation in a variety of different ways, including numerically, algebraically, graphically, physically, in words; and to interpret, analyse, and compare such representations
- U8 apply their knowledge and skills to solve a problem, including decomposing it into manageable parts and/or simplifying it using appropriate assumptions
- U11 generate general mathematical statements or conjectures based on specific instances

The task is linked to the following contextual strand learning outcomes from the Junior Cycle Mathematics specification:

- GT2 investigate 2D shapes so that they can:
 - c) find the perimeter and area of plane figures made from combinations of discs, triangles, and rectangles, including relevant operations involving pi
- AF1 investigate patterns and relationships so that they can:
 - a) represent these patterns and relationships in tables and graphs
- AF2 investigate situations in which letters stand for quantities that are variable

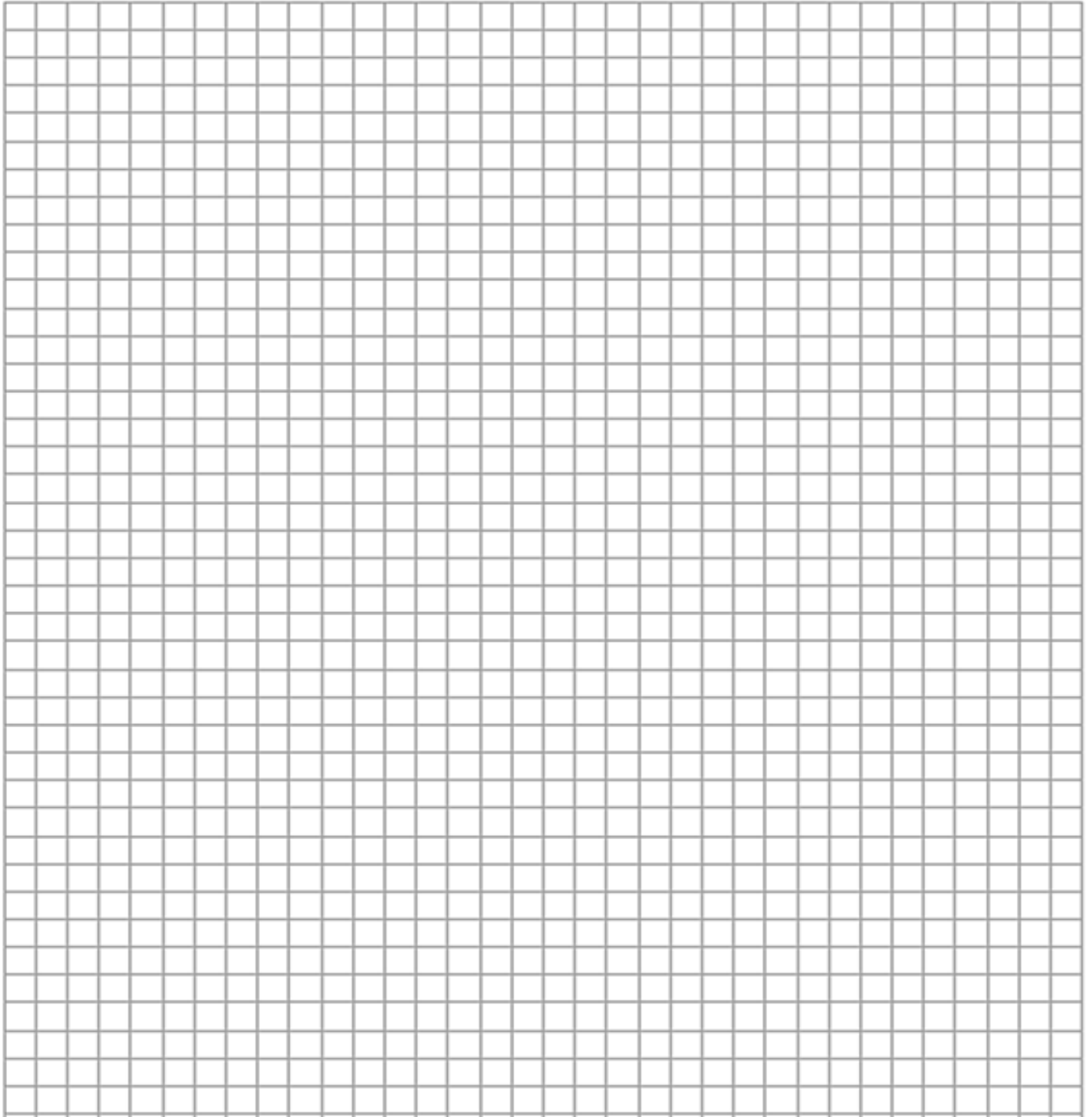
It may be necessary to reduce the difficulty level of the task. Planned questioning may be useful to guide students towards a generalisation or mathematical argument. It is recommended, however, that scaffolding be kept to a minimum to allow students to apply their knowledge in unfamiliar situations and create a need for the construction of new knowledge. If the task is scaffolded students may engage with the task multiple times over the three years of Junior Cycle.

The task allows for the reinforcement of students' knowledge and understanding from the relevant aspects of the Shape and Space strand of the primary school curriculum, while simultaneously providing an opportunity for extension and enrichment for students who may have fully grasped these concepts before commencing first year in post-primary. A Quick Reference Guide to the Links between the Primary and Post-Primary Curricula can be found [here](#).

Please see below for possible extension questions/tasks that may provide an opportunity for deeper learning for students.






'If a piece is cut off a two-dimensional shape it reduces both the area and the perimeter.'

Investigate this statement.
What conclusions can be drawn?
Justify your reasoning.



Possible prompts or extension tasks / questions for student

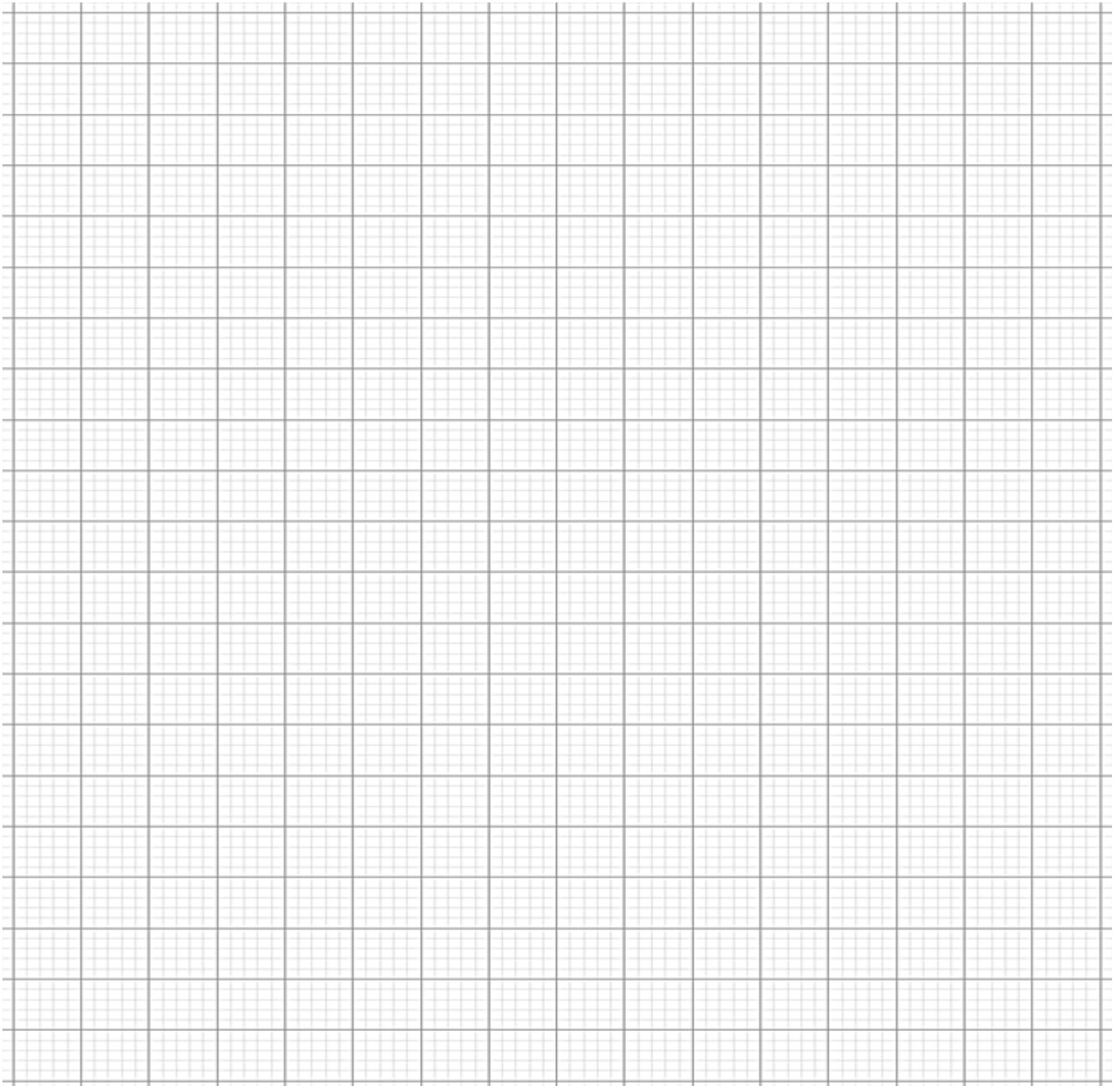
Please note: Measurements not included – prompts only

Possible prompts for triangles	Further questions
	<p>When a piece is cut off this shape (perpendicular to the base) you:</p> <ol style="list-style-type: none"> Reduce its area Reduce its perimeter <p>Investigate this statement. What conclusions can be drawn? Justify your reasoning.</p>
	<p>When a piece is cut off this shape (parallel to a side) you:</p> <ol style="list-style-type: none"> Reduce its area Reduce its perimeter <p>Investigate this statement. What conclusions can be drawn? Justify your reasoning.</p>
Possible prompts for quadrilaterals	Further questions
<p>Square</p> 	<p>What happens to the area and perimeter of a square when a piece is cut off the shape?</p> <p>Can your observations be generalised?</p> <p>What happens to the area and perimeter when a section is cut from inside the square?</p>
<p>Rectangle</p> 	<p>What happens to the area and perimeter if a triangle is cut off a rectangle?</p>
<p>Rectangle</p> 	<p>What happens to the area and perimeter if a rounded section is cut from a rectangle?</p> <p>What happens to the area and perimeter if a series of rounded sections are cut from the rectangle's edges?</p>

Fencing an Enclosure Task

Emily has 20 metres of fencing. She needs to form an enclosure in the garden for her dog. What is the maximum area she can enclose if all the fencing must be used?

Justify your answer.



TRIANGLES

Right Angled Triangle:

$$a = 5, b = 6\frac{2}{3}, c = 8\frac{1}{3}$$

$$\text{Area} = 16\frac{2}{3} \text{ Units}^2$$

Isosceles Triangle:

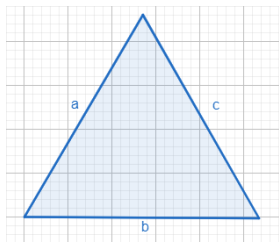
$$a = 6, b = c = 7$$

$$\text{Area} = 18.97 \text{ Units}^2$$

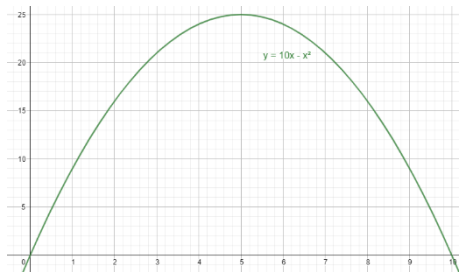
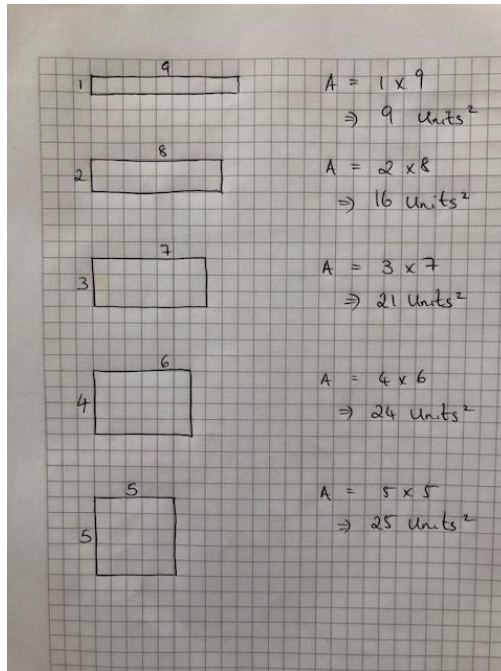
Equilateral Triangle:

$$a = b = c = 6\frac{2}{3}$$

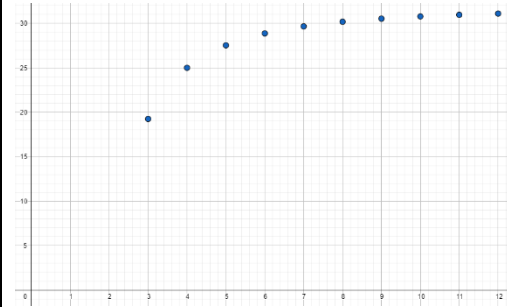
$$\text{Area} = 19.25 \text{ Units}^2$$



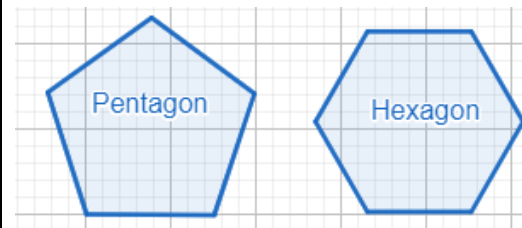
QUADRILATERALS



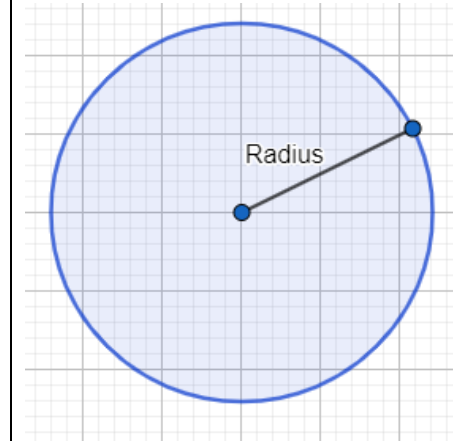
OTHER POLYGONS



Perimeter	Sides	Area
20	5	27.53
20	6	28.87
20	7	29.66
20	8	30.18
	.	
	.	
	.	
20	20	31.57



CIRCLE



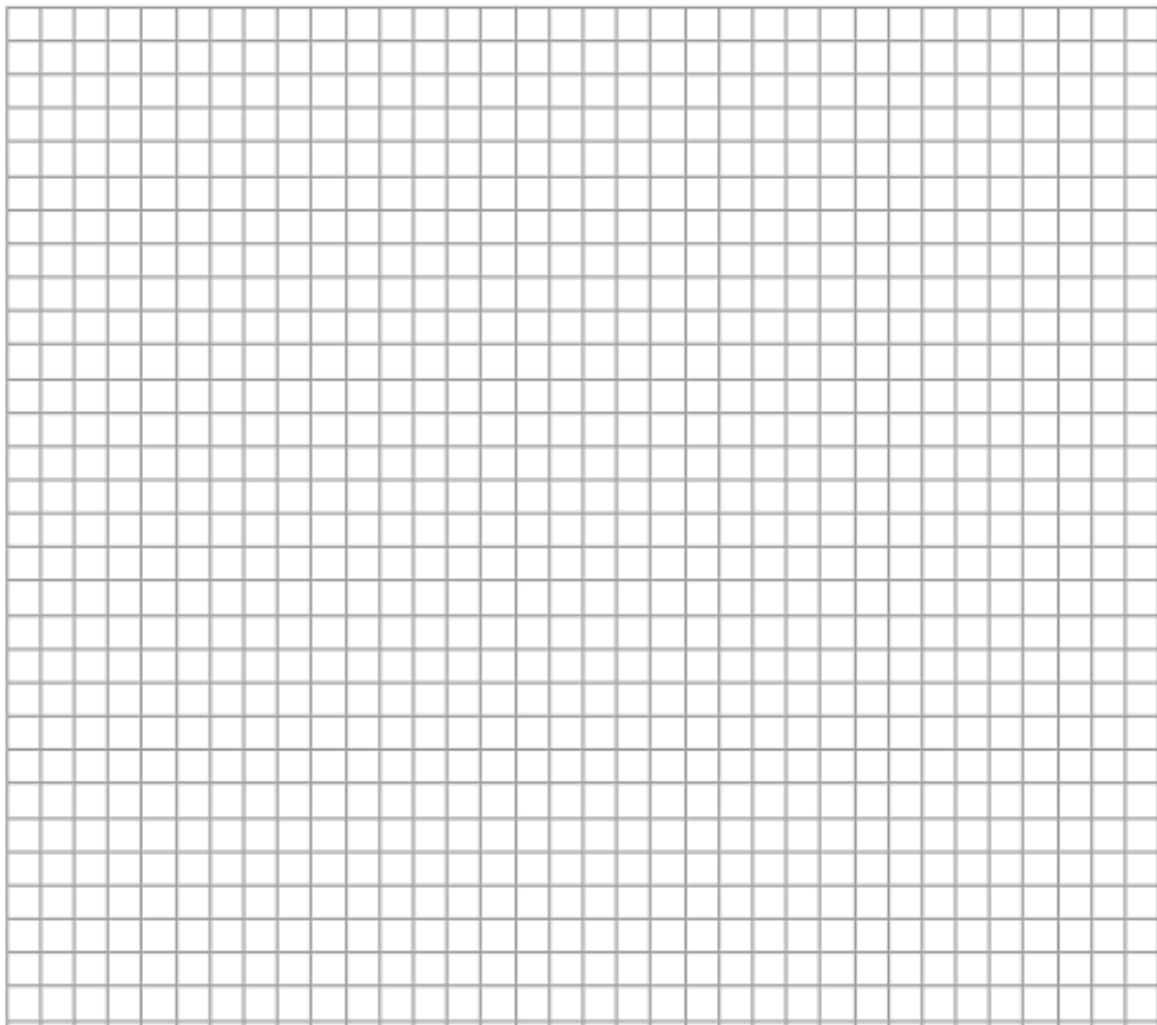
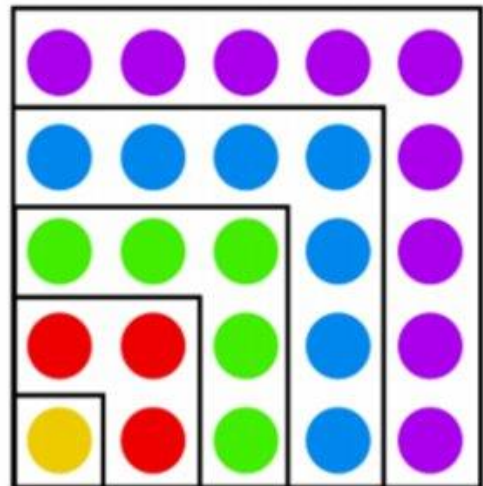
$$\begin{aligned} \text{Circumference} &= 20 \\ 2\pi r &= 20 \\ \pi r &= 10 \\ r &= \frac{10}{\pi} \\ \text{Area} &= \pi r^2 \\ \text{Area} &= \frac{100}{\pi} \\ \text{Area} &\approx 31.83 \text{ Units}^2 \end{aligned}$$

Coloured Dots

What questions do you have about the image?

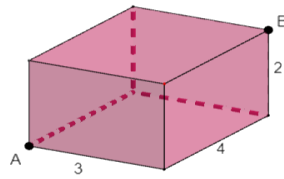
What do you want to know about the image?

Is there something that you would like to research or investigate based on the image?

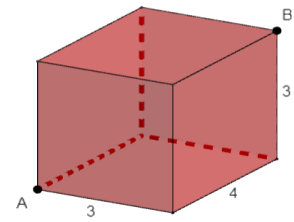


Cuboid Task

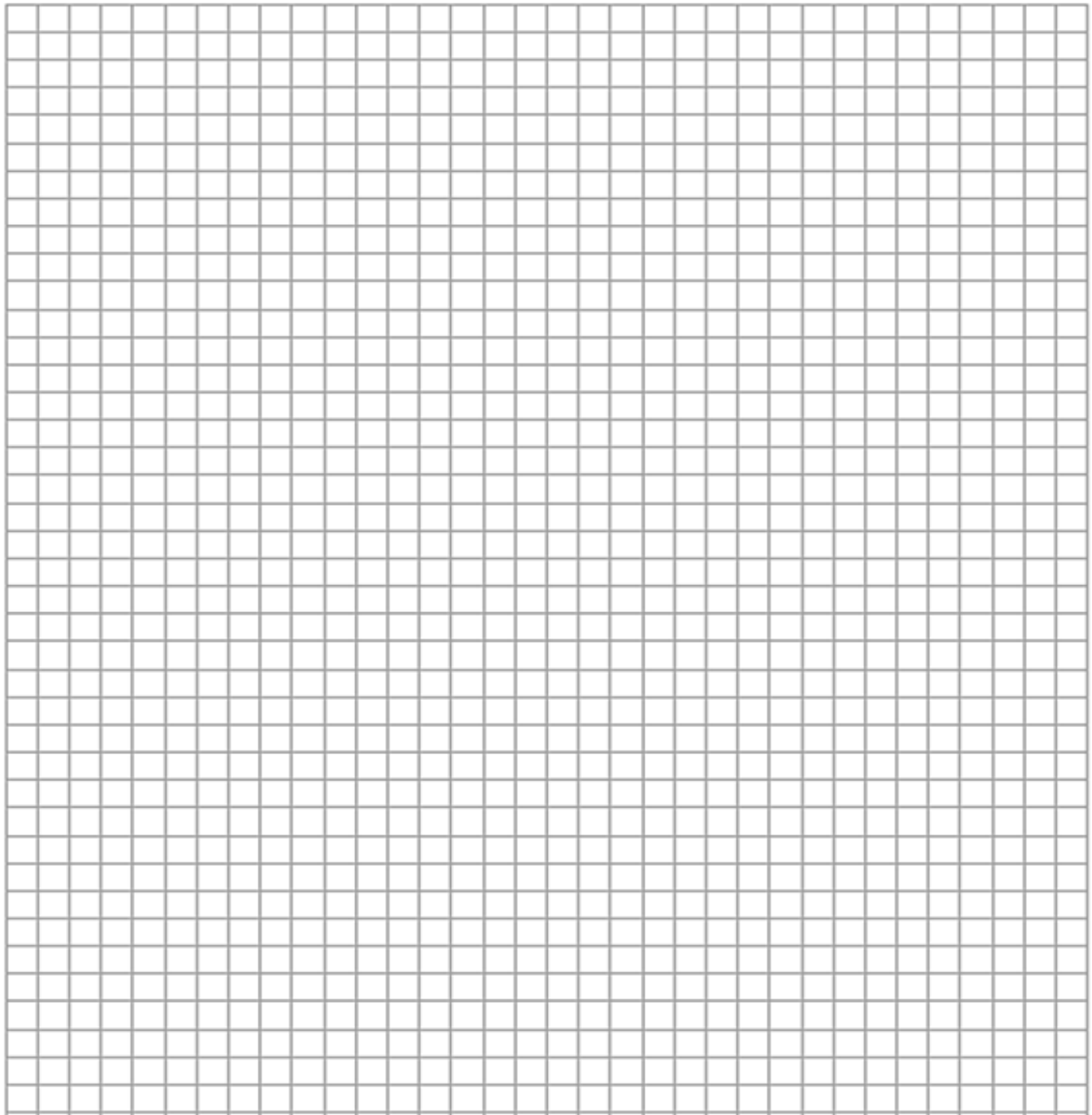
By imagining an ant walking on the cuboids shown in the diagram, find the shortest path for the ant to walk from A to B along the surface.

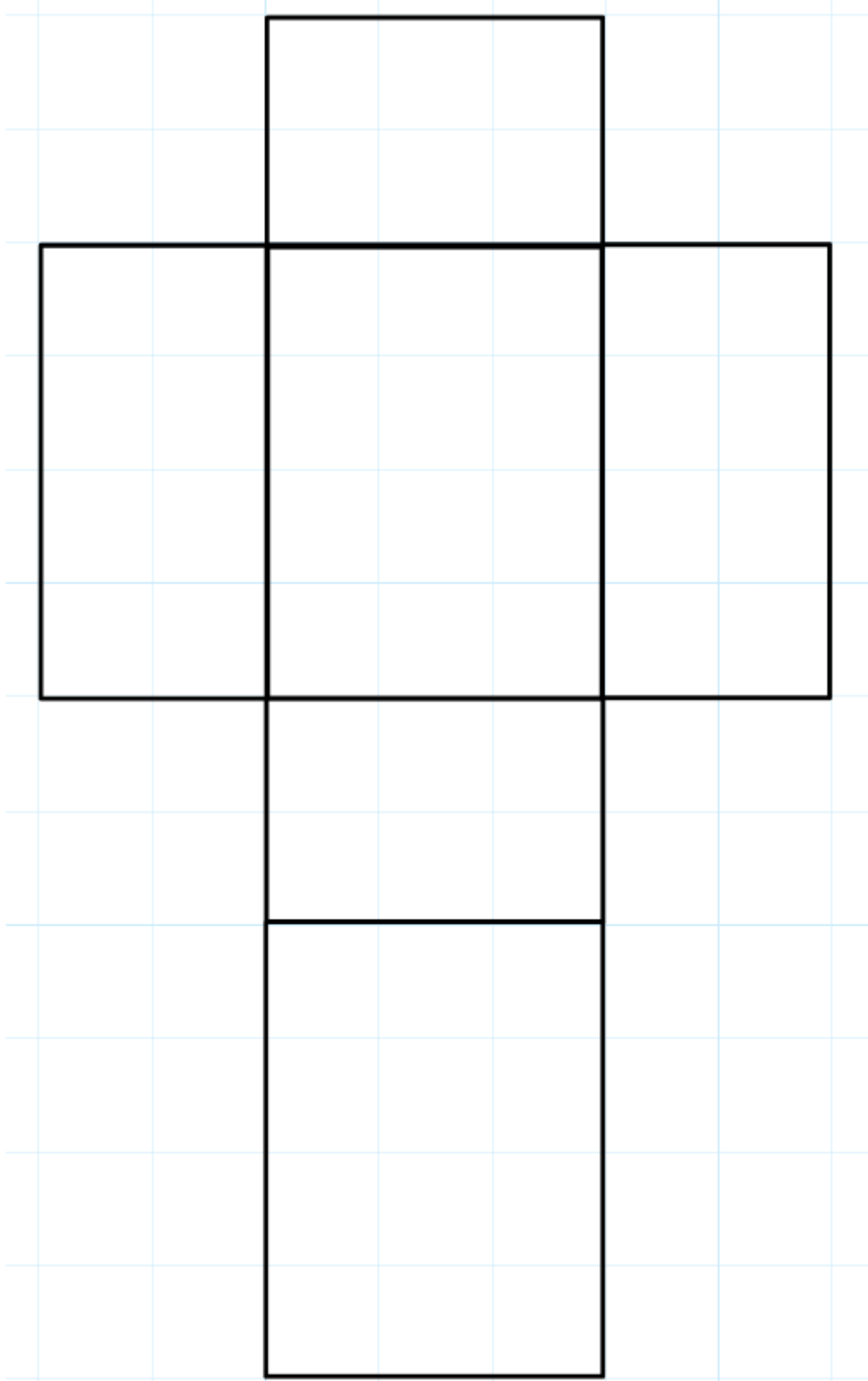


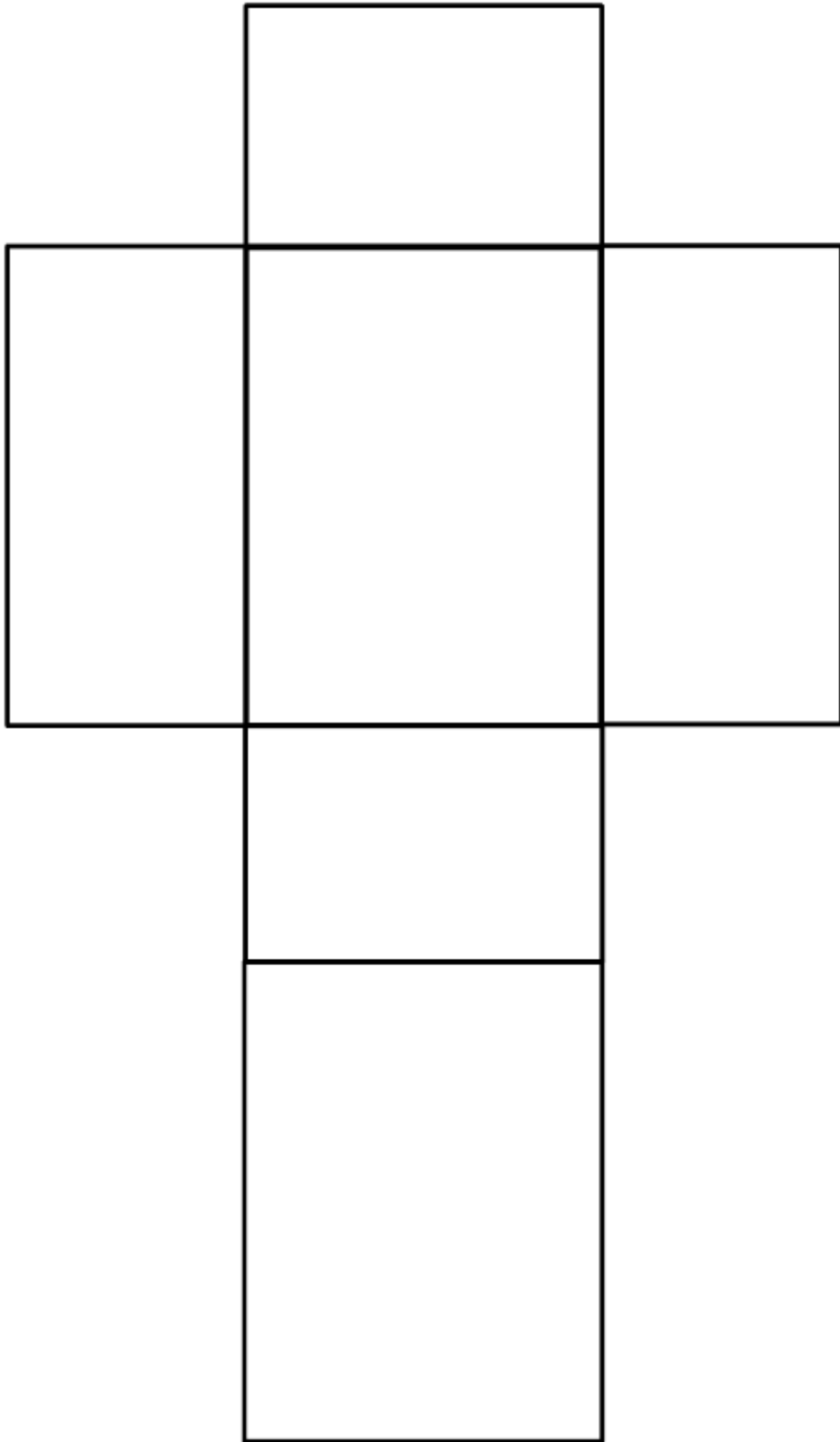
Cuboid



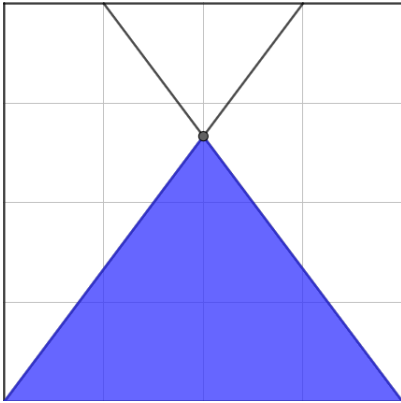
Cuboid



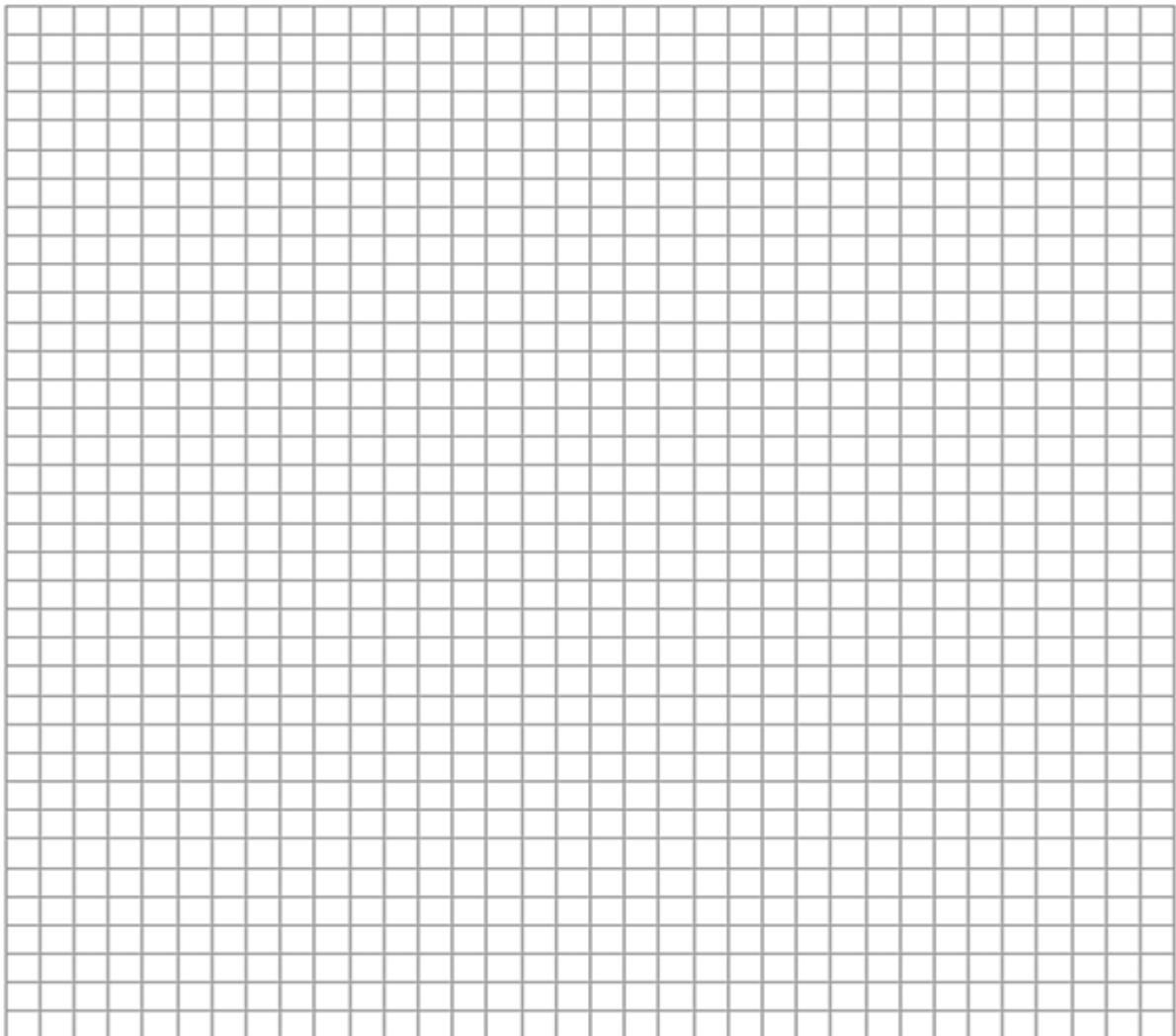




Fraction of Shape



What fraction of the shape is shaded?

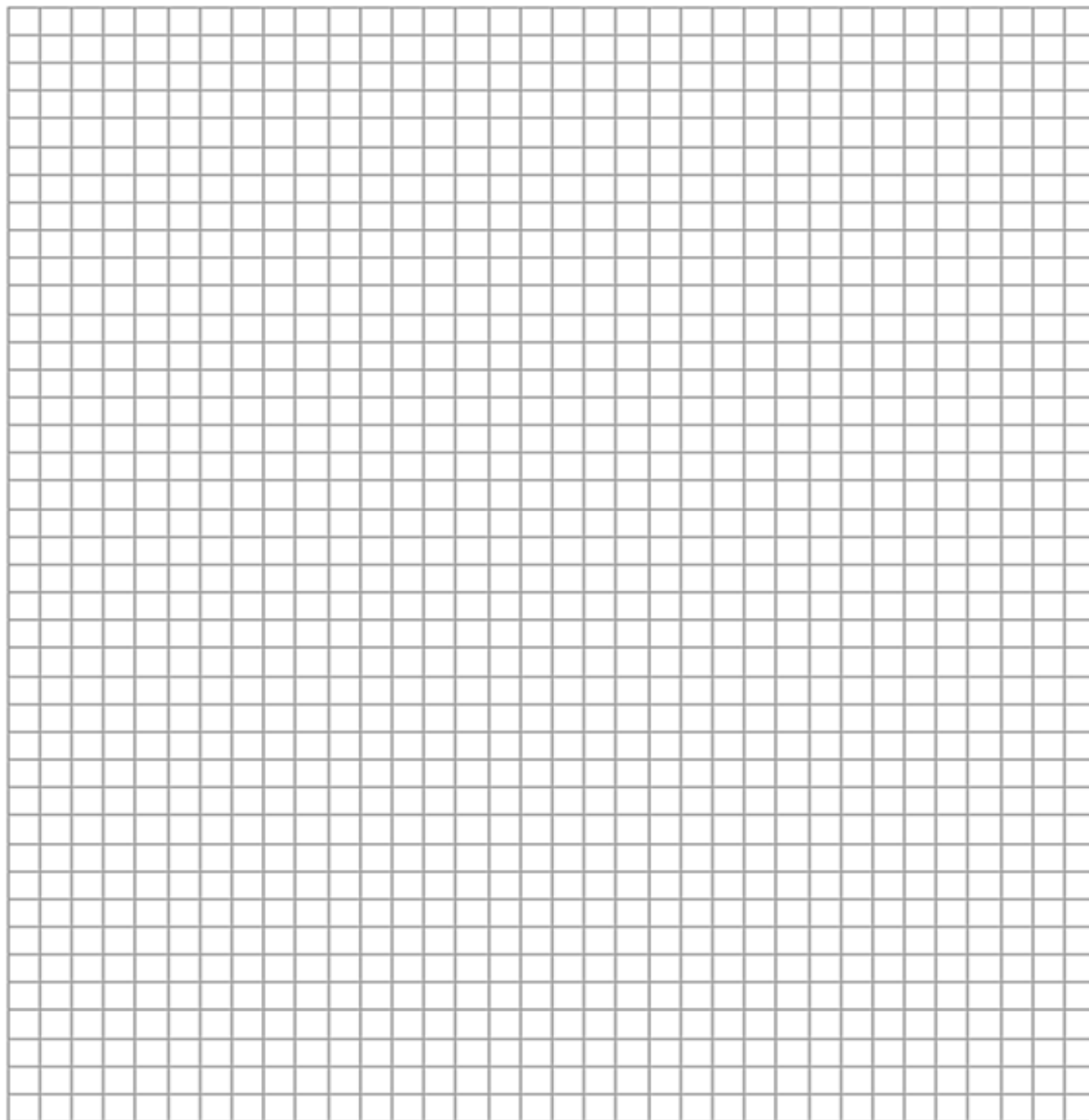


Mathematical Argument

Consider the following conjecture:

$$x^2 + 1 \text{ can never be } 0.$$

Use a mathematical argument to convince someone else why the conjecture is either true or false.



Painted Cube

Painted Cube

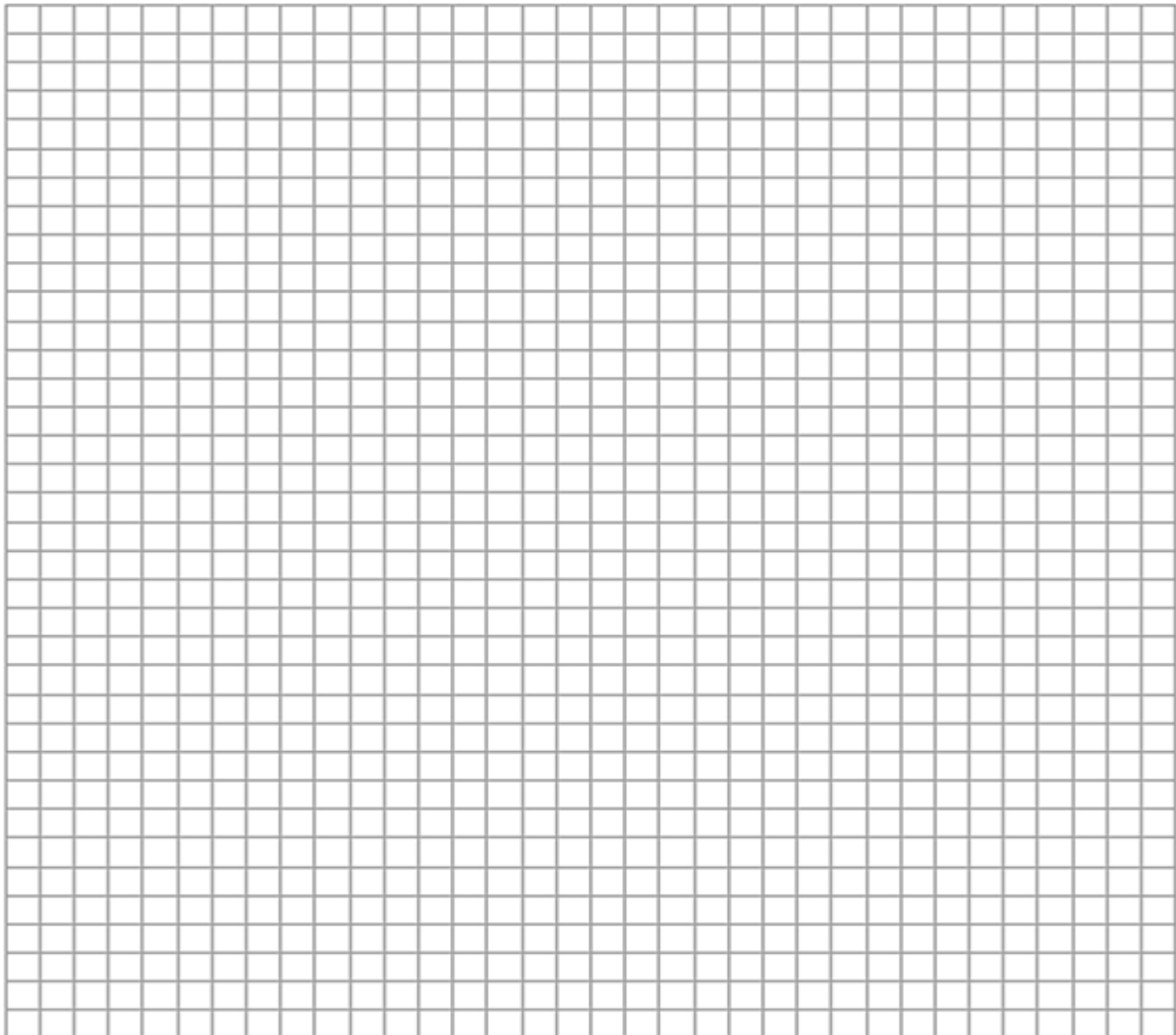
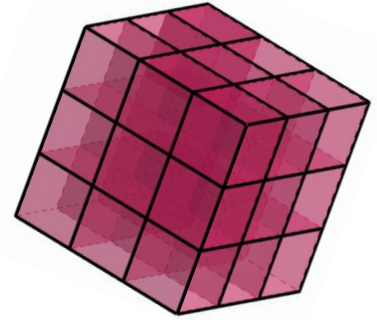
A $3 \times 3 \times 3$ cube made up of 27 $1 \times 1 \times 1$ cubes, is dipped in a bucket of paint. After the paint has dried, the $3 \times 3 \times 3$ cube is taken apart into its 27 individual cubes.

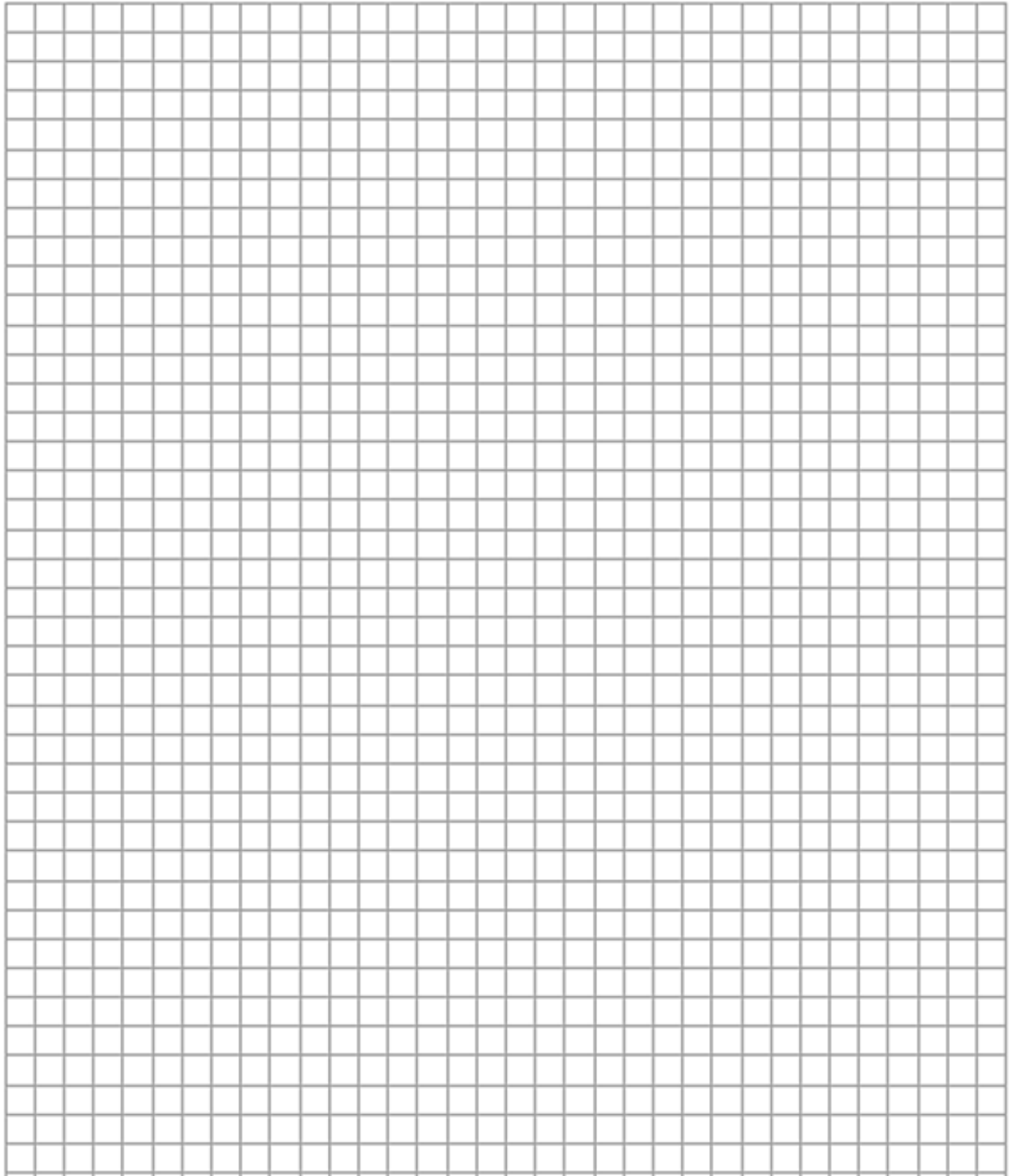
How many of these individual cubes have paint on three sides?

What if it were a $4 \times 4 \times 4$ cube?

What if it were a $5 \times 5 \times 5$ cube?

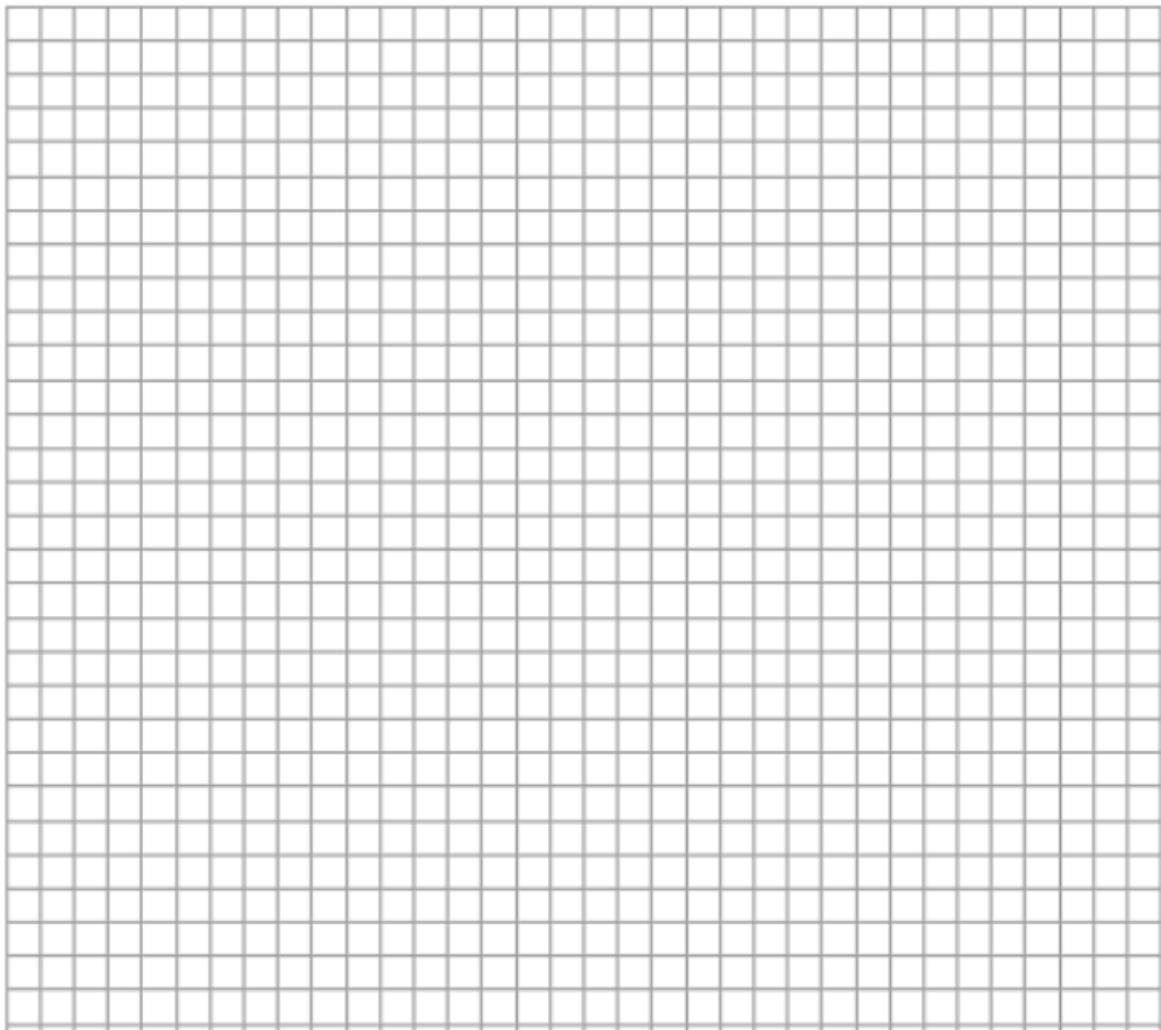
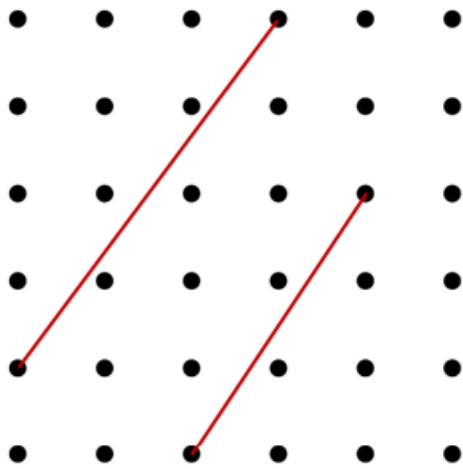
What if it were an $n \times n \times n$ cube?



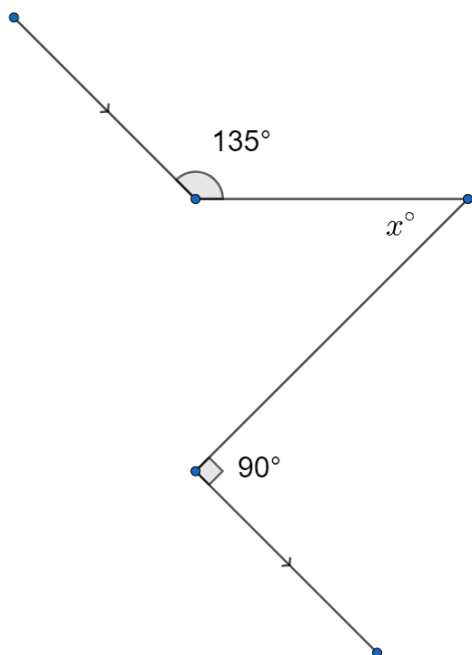


Parallel Lines

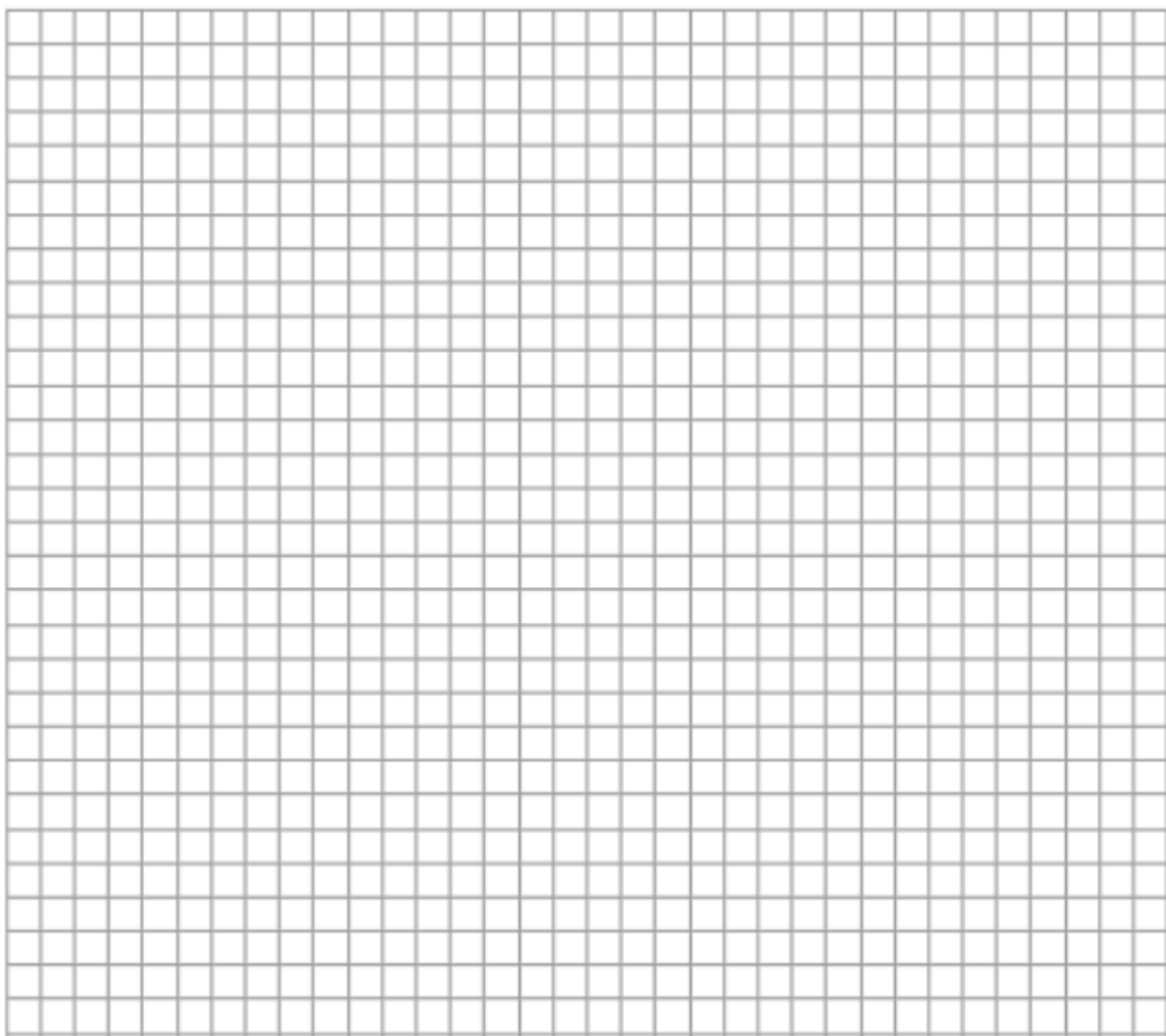
Are these lines parallel?
Justify your reasoning



Size of Angle



Find the size of the angle labelled x ?



Algebraic Statements

Sort each of the following 8 algebraic statements into the categories:

Always True, Sometimes True, and Never True

$$x + 5 \text{ is less than } 20$$

$$6x = 2(3x)$$

$$x^2 = 5$$

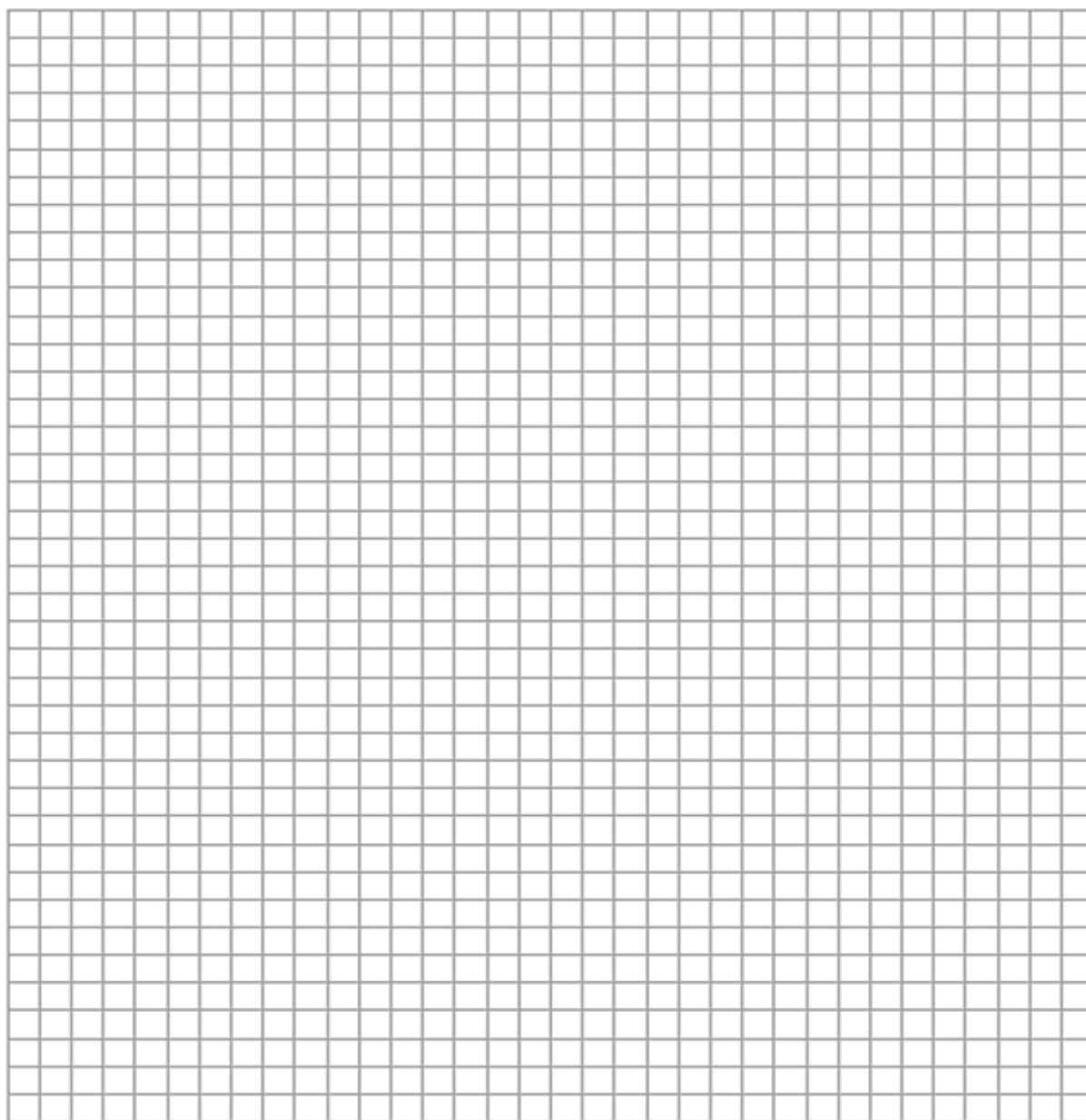
$$x + 2 = x + 16$$

$$(x)(x) = 2x$$

$$9x^2 = (3x)^2$$

$$2x + 3 = 3 + 2x$$

$$x^2 = 9$$

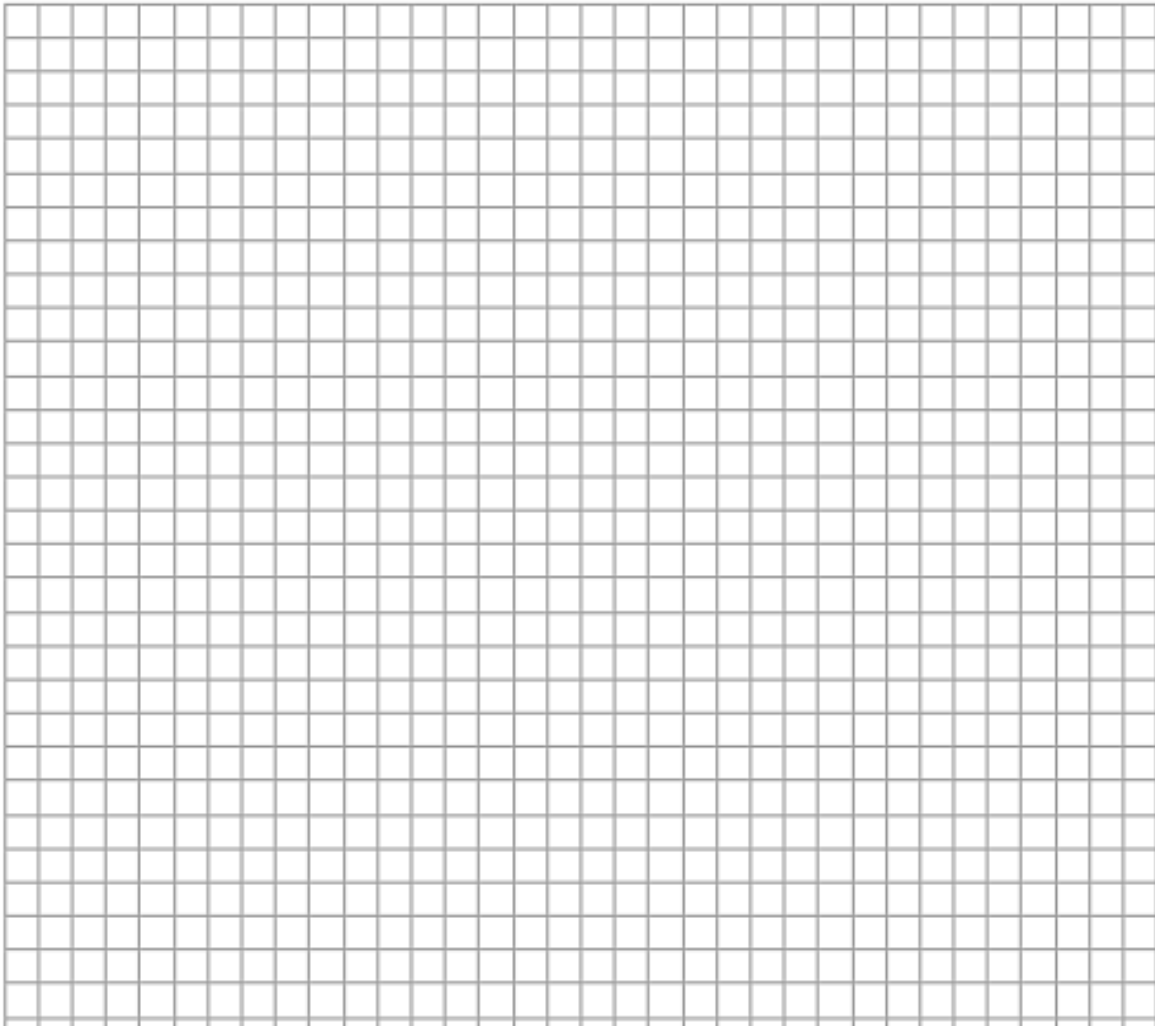


How to work your glove size out.

Measure around the largest part of your hand, passing over your knuckles, whilst making a fist, but exclude the thumb



Glove Size	Small	Medium	Large	X Large
Hand Size (cms)	18 - 21.5	20 - 24	23 - 26.5	25.5 - 29
Hand Size (inches)	7 - 8.5	8 - 9.5	9 - 10.5	10 - 11.5



Is there a process we can use to find the mean, median or modal hand size for a student within the class group?

Consider the following:

- How can we **measure** everyone's hands?
- How can we **collect** this data?
- How can we **compare** the data we collect?
- How can we **analyse** the data we collect?

Group Work

How are you going to take the measurement(s)?
How are you going to record the data?

What is your group's statistical question?
What measurement(s) are you going to take?



What calculations are you going to perform?
How are you going to display the data?

Do my calculations and display help me answer the question?
Is there another calculation that could help or possibly change my answer?

Don't forget to check the success criteria!

CSO Website

The following areas of the CSO Website, provide teachers with authentic data that can be used in class, to support students' learning of statistics

Home Statistics Databases Methods About Us Census



-  **4.8%**
Monthly Unemployment rate
February 2020
-  **↓ 3.4%**
Retail Sales Index
January 2020
-  **↑ 1.3%**
Consumer Price Index
January 2020
-  **↑ 0.9%**
Residential Property Price Index
December 2019
-  **↑ 1.7%**
Real GDP
Quarter 3 2019

News and Events

- Census 2021 consultation on outputs **NEW**
- Seminar on Sector Accounts and Globalisation **NEW**
- CSO to Oversee New National Survey on the Prevalence of Sexual Violence in Ireland **NEW**
- Interactive tool - Baby Names of Ireland
- Events, Conferences & Seminars

[view more news >](#)

Taking Part in a Survey?

CSO Household Surveys

[go to taking part in a survey >](#)

Retail Sales Index - Seasonally Adjusted Volume



Latest Releases






- Industrial Production and Turnover Postponed
5 March 2020
- Live Register February 2020
5 March 2020
- Output, Input and Income in Agriculture - Preliminary Estimate 2019
5 March 2020
- Monthly Service Index January 2020
4 March 2020
- Monthly Unemployment February 2020
3 March 2020

[more releases >](#)

[release calendar >](#)

[press releases >](#)

Statistics

- People & Society 
- Labour Market & Earnings 
- Business Sectors 
- Economy 
- Environment & Climate 

CSO Visualisation Tools



**BREXIT INDICATORS:
IRELAND AND THE UK IN NUMBERS**

INBOUND TRAVEL, TRADE SERVICES, OUTBOUND TRAVEL, TRADE GOODS, MIGRATION

Brexit Indicators: Ireland and the UK in Numbers

[Visualisation Tools >](#)

[Infographics >](#)

Most Popular Names by Year of Birth

On the homepage select **Visualisation**

Tools



Select the tile for **Baby Names of Ireland** and follow the prompts



Using a SAPMAP

1) On the homepage select **Visualisation**

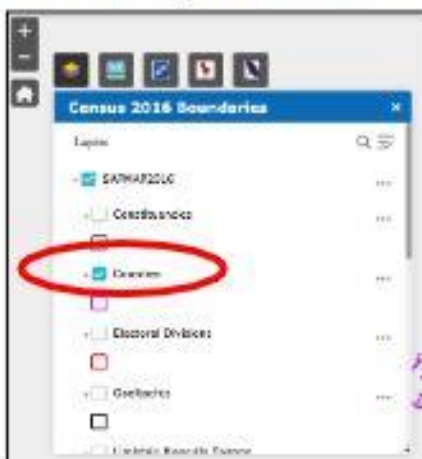
Tools



2) Select **SAPMAP**



3) Deselect the option for counties



4) Select **Small Areas** and **Zoom** to find your area



- 5) Select your area on the map and select the option for **more information**



- 6) Select one of the theme options such as **Motor Car Availability, PC Ownership and Internet Access**



- 7) Select one of the available links for a detailed breakdown for your local area



PDF Excel Print

Number of households with internet access

Internet Access	Households
Broadband	82
Other	9
No	11
Not stated	7
Total	109

- 8) Select **Compare state total data** to make a comparison with national figures



PDF Excel Print

Number of households with internet access

Internet Access	Households
Broadband	1,200,067
Other	161,676
No	312,582
Not stated	52,340
Total	1,697,665

Infographics

On the homepage select **Infographics**



Select a theme



Student Resource - Problem Solving Toolkit

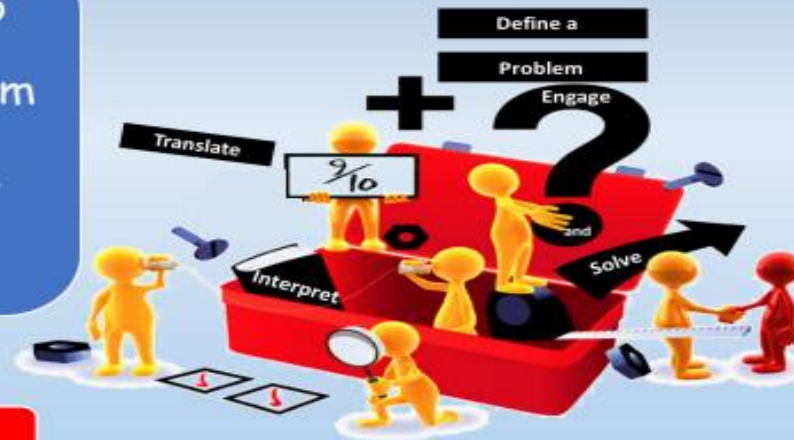
Students can draw on aspects of the Problem-Solving Toolkit where appropriate as a support when engaging with various learning experiences across Junior Cycle such as carrying out investigations, engaging with tasks and solving problems

An tSraith Shóisearach do Mhúinteoirí
JuniorCYCLE
for teachers

Problem-Solving Toolkit

Define a problem

What do I want to know? What do I already know? Form a conjecture, specialise/reduce, restrict/extend, stress/ignore



Engage and Solve

Spatial/Graphical Reasoning, List, Draw, Work backwards, Eliminate, Sort, Classify, Estimate, Compare, Organise, Vary, Try extreme cases, Work systematically

Translate

Tools, Concrete Materials, Tables, Diagrams, Formulae, Check for hidden assumptions

Interpret

Summarise, Explain, Justify, Reason, Generalise, Prove, Judge the Validity, Convince, Predict, Draw conclusions, Mathematical Solutions, Real World Solutions.

Student Resource - Statistical Enquiry Cycle

Students can draw on aspects of the Statistical Enquiry Cycle where appropriate as a support when engaging with various learning experiences across Junior Cycle such as designing and carrying out investigations and organising, managing and analysing data



IDENTIFY VARIABLES AND SELECT RELIABLE METHODS FOR GATHERING DATA

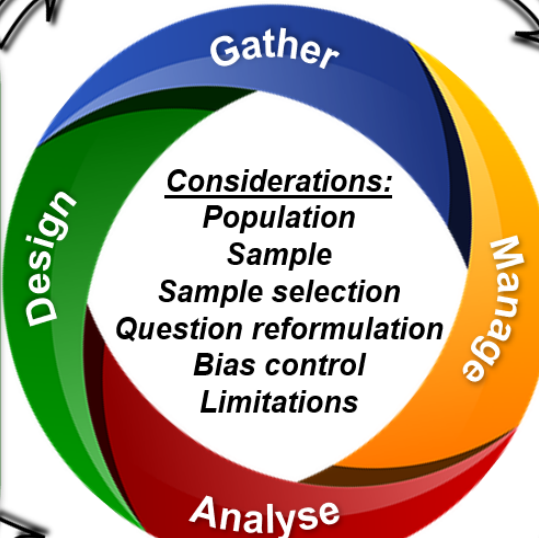
- What variable(s) are being measured?
- How are the variable(s) being measured?
- Is the sample representative of the population?



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JuniorCYCLE
 for teachers

INVESTIGATION DESIGN

- Is the statistical question posed concise?
- What are the variable(s) that may need to be measured to answer the question?
- Is it possible to collect the data that can answer the question?
- Is it possible to draw down or gather the required data in a given timeframe?
- Is an experiment, survey or observational study being conducted?



ORGANISE AND MANAGE DATA

- Is the data displayed in a table, diagram, chart and/or graph?
- Is the data summarised numerically, graphically, diagrammatically and/or with words?
- Is the display/summary method most appropriate?



ANALYSE AND INTERPRET DATA

- Is the data, display and/or summary connected to the statistical question?
- How could the investigation be improved?

