Supporting Students in Developing their Ability to Pose Questions and Formulate Problems in Junior Cycle Mathematics

Self-Directed Interactive CPD

JCT Mathematics

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An tSraith Shóisearach do Mhúinteoirí
Junior CYCLE for teachers

Introduction

Supporting Students in Developing their Ability to Pose Questions and Formulate Problems in Junior Cycle Mathematics is a self-guided interactive piece of CPD. This CPD has been designed to give junior cycle mathematics teachers the opportunity to consider how they can enable students to develop their question posing and problem formulation abilities.

The anticipated time to complete this CPD is 45 minutes.

The learning intentions are:

- to consider the benefits of integrating question posing and problem formulation into pedagogical practice
- to engage with classroom strategies that can be used to incrementally develop students' ability to pose their own mathematical questions and problems from first year onward
- to consider ways in which to support students when defining problem statements as part of Mathematical Investigation (CBA 1)

There are three sections for this CPD. Each of the three sections has a separate Google Form. To access the Google Forms, visit: https://forms.gle/j95Ur6qrwUg7LP7i8

Using this Booklet

This booklet should be used along with the three interactive Google Forms which can be accessed using the link found in the introduction.

Each Form includes some activities for you to engage with as you progress through each section. If engaging with this CPD with colleagues, you may wish to discuss and share your ideas as part of the various activities. If engaging with this CPD on an individual basis, you may wish to take time to document your thoughts as you progress through each section. Space is provided in this booklet for you to capture notes based on your thoughts and/or discussions.

If you are using the **printed PDF version** of this booklet you can document your thoughts and ideas by *writing* in the spaces provided.

If you are using the **editable Word version** of this booklet you can document your thoughts and ideas by *typing* in the spaces provided.

Section One: Problem Posing, Question Posing and Problem Statements

This video/audio features Dr Paddy Johnson, lecturer in mathematics education at the University of Limerick, discussing problem posing, question posing and problem statements in mathematics education

Ctrl + Click on the image to watch the video on YouTube.

Use this space to take notes or record your thoughts about problem posing, question posing and problem statements.

Section One: Read, Summarise and Reflect

Read	Choose **one** of the three extracts on pages 4 and 5 which discuss question/problem posing. Take some time to read through the extract.
Summarise	Record what you consider to be the main point(s) of the extract.
Consider or Discuss	If working **as an individual**, take some time to reflect on the extract and record your thoughts?
	If working **as a group,** take some time to discuss each of the extracts and record your collective thoughts.

Extract 1

Problem posing is an important component of the mathematics curriculum, and is considered to be an essential part of mathematical doing (Brown & Walter, 1993, NCTM, 2000). Problem posing involves generating of new problems and questions aimed at exploring a given situation as well as the reformulation of a problem during the process of solving it (Silver, 1994). Providing students with opportunities to pose their own problems can foster more diverse and flexible thinking, enhance students' problem solving skills, broaden their perception of mathematics and enrich and consolidate basic concepts (Brown & Walter, 1993, English, 1996). In addition, problem posing might help in reducing the dependency of students on their teachers and textbooks, and give the students the feeling of becoming more engaged in their education. Cunningham (2004) showed that providing students with the opportunity to pose problems enhanced students' reasoning and reflection. When students, rather than the teacher, formulate new problems, it can foster the sense of ownership that students need to take for constructing their own knowledge. This ownership of the problems results in a highly level of engagement and curiosity, as well as enthusiasm towards the process of learning mathematics.

Lavy, I. and Shriki, A., 2007. Problem posing as a means for developing mathematical knowledge of prospective teachers. In *Proceedings of the 31st Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 129-136). Seúl: PME.

Extract 2

Like problem solving, problem posing can be viewed as a classroom activity. There is increased interest in integrating mathematical problem-posing into classrooms. For example, Silver (1997) proposed that engaging students in problem posing can foster their creativity. There are at least two reasons why engaging students in problem-posing activities is likely to have a positive impact on their learning. First of all, problem-posing activities are usually cognitively demanding tasks. Doyle (1983) argues that tasks with different cognitive demands are likely to induce different kinds of learning. Problem-posing activities in classrooms have the potential to provide intellectual contexts for students' rich mathematical development. Such activities can promote students' conceptual understanding, foster their ability to reason and communicate mathematically, and capture their interests and curiosity (NCTM, 1991). Second, problem-solving processes often involve the generation and solution of subsidiary problems (Polya, 1957).

Silver, 1994 suggests that student-posed problems are more likely to connect mathematics to students' own interests, something that is often not the case with traditional textbook problems. Thus, encouraging students to generate problems is not only likely to foster student understanding of problem situations, but also to nurture the development of more advanced problem-solving strategies.

Singer, F.M., Ellerton, N., Cai, J. and Leung, C.K.E., 2011. Problem posing in mathematics learning and teaching: A research agenda.

Extract 3

The benefits of posing problems have been pointed out by many researchers in mathematics education. For instance, Pittalis et al (2004) pointed out that students could solve mathematical word problems as a result of using problem posing as an instructional strategy. Similarly, English (1997) found that problem posing afforded teachers the opportunity to comprehend students' thinking about concepts and processes in mathematics. Research has also indicated that problem posing improves students' thinking and problem solving skills, attitudes and confidence in mathematics (Guvercin et al 2014). Similarly, posing problem fosters flexible thinking, and enhances students' problem solving skills, reasoning and reflection. Problem posing is a necessary part of learning mathematics. Kojima et al (2015) say it is an important activity in mathematics education, and lies at the heart of mathematical activity and that problem posing is a necessary skill for solving problems. Singer et al (2013) assert that posing problem raises creativity in students and adds to mathematics talent and independent learning.

It could be said with every confidence that, if properly handled, problem posing instructional strategy can be one of the most effective strategies in the teaching and learning of mathematics. This is because not only students benefit from problem posing; but teachers also do. Problem posing affords teachers the opportunity to get an insight into students' thinking processes. Problem posing revealed that mathematics anxiety and fears could be reduced significantly in students. It encourages the use of diverse representations, building of a knowledge network, development of creativity, enhancement of attitude toward mathematics and increment in self-confidence. It can safely be advocated that problem posing be an integral part of the mathematics curriculum, considering the role it can play in the teaching and learning of the subject.

Adapted from Zuya, Habila Zuya, 2017. The Benefits of Problem Posing in the Learning of Mathematics: A Systematic Review. In *International Journal of Advanced Research*. (pp. 853-860).

Section Two: Question Adaptation

Activity

Take a moment to read the question below.

Consider how this question could be adapted.

A running track is made up of two semi-circles with radius 20 metres and two straights with length 100 metres.

Find the distance round one lap of the track.

[Task sourced from Swan and Burkhardt (2012)]

Use this space to record some of your adapted question suggestions.

Some possible approaches to adapting questions are given in the text box on the right.

Some possible ways that students might approach adapting questions are by:

- Adding, removing or changing a condition of the original question
- Change or add context
- Re-word the question

Note: not all alternatives are applicable to all questions

Section Two: Question Posing using an Image as a Prompt

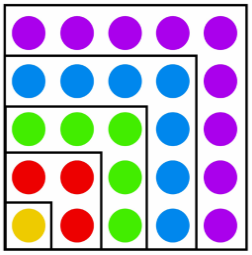
Choose one of the three image prompts to use for this question posing strategy.

Take a few minutes to examine the image, then consider the prompt questions below:

What questions do you have?

What do you want to know about?

Is there something that you would like to research or investigate?

Base Jumping	Vertical Forrest	Coloured Dots
	 Attribution	 Attribution
Use this space to record your questions.		

Section Two: Question posing using a video as a Prompt

Choose **one** of the three video prompts to use for this question posing strategy.

Take a few minutes to watch the video, then consider the prompt questions below:

What questions do you have?

What do you want to know about?

Is there something that you would like to research or investigate?

3 – Pointer Game Shot	**Vertical Farming**	**Infinite Patterns**
		

Use this space to record your questions.

Section Two: Question posing using GeoGebra

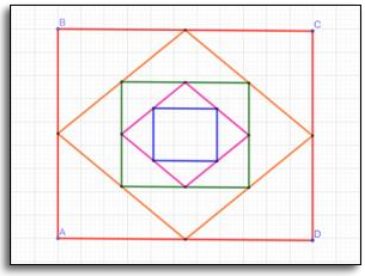
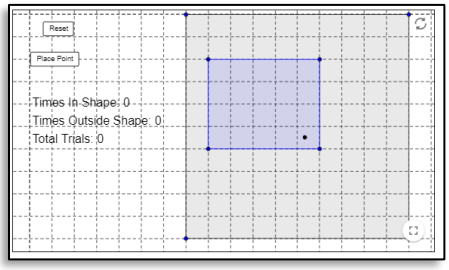
Choose **one** of the two GeoGebra files to use for this question posing strategy.

Take a few minutes to examine the GeoGebra file, then consider the prompt questions below:

What questions do you have?

What do you want to know about?

Is there something that you would like to research or investigate?

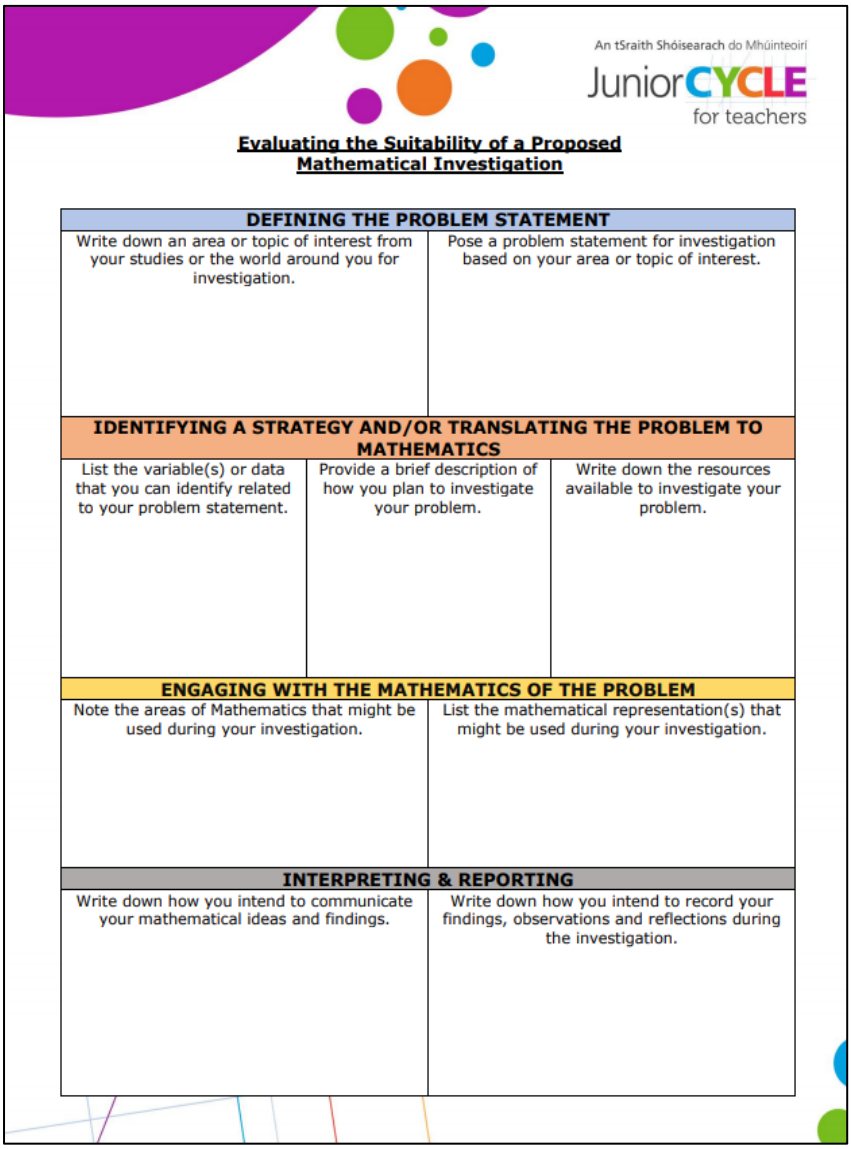
Rectangles in Rectangles	**Probability and Shape**
	

Use this space to record your questions.

Section Three: Defining Problem Statements for Mathematical Investigation (CBA 1)

What different classroom activities could you plan such that students have opportunities to develop the necessary skills for Mathematical Investigation?

(You may find the classroom resource pictured below useful when considering this question. The full resource can be accessed using the link https://bit.ly/3iYJaht.)



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Evaluating the Suitability of a Proposed Mathematical Investigation

DEFINING THE PROBLEM STATEMENT	
Write down an area or topic of interest from your studies or the world around you for investigation.	Pose a problem statement for investigation based on your area or topic of interest.

IDENTIFYING A STRATEGY AND/OR TRANSLATING THE PROBLEM TO MATHEMATICS		
List the variable(s) or data that you can identify related to your problem statement.	Provide a brief description of how you plan to investigate your problem.	Write down the resources available to investigate your problem.

ENGAGING WITH THE MATHEMATICS OF THE PROBLEM	
Note the areas of Mathematics that might be used during your investigation.	List the mathematical representation(s) that might be used during your investigation.

INTERPRETING & REPORTING	
Write down how you intend to communicate your mathematical ideas and findings.	Write down how you intend to record your findings, observations and reflections during the investigation.

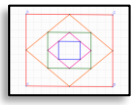
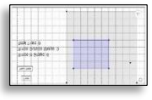
Reflection Activity: My Action Points

Take some time to reflect on your learning from this piece of CPD.

What opportunities might I provide students with to support them in developing their ability to pose questions and formulate problems in mathematics?

Resources and Supports

In the self-directed CPD *Supporting Students in Developing their Ability to Pose Questions and Formulate Problems in Junior Cycle Mathematics* a number of resources were featured to support students in developing their question posing and problem formulation abilities. These resources are listed below for your convenience with links to view or download each resource individually.

Resources for the development of student's question posing abilities.		
The following questions may be used to support formative assessment and formative feedback for students: What questions do you have? What do you want to know about? Is there something that you would like to research or investigate?		
Question Posing using an Image as a Prompt		
1.	Base Jumping File Type: Image	
2.	Vertical Forrest File Type: Image	
3.	Coloured Dots File Type: Image	
Question Posing using a Video as a Prompt		
4.	3-Pointer Game Shot File Type: YouTube	
5.	Vertical Farming File Type: YouTube	
6.	Infinite Patterns File Type: YouTube	
Question Posing using Virtual Manipulatives as a Prompt (GeoGebra)		
7.	Rectangles in Rectangles File Type: GeoGebra	
9.	Geometric Probability File Type: GeoGebra	

Further Resources for Problem Posing and Question Formulation		
1.	Cube Task File Type: PDF	
2.	Post Box Task File Type: PDF	
3.	Evaluating a Proposed Mathematical Investigation File Type: PDF