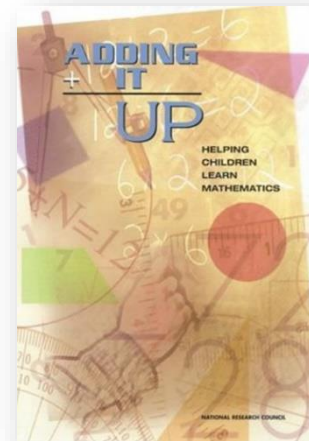


## The Components of Mathematical Proficiency

### Productive Disposition

*Productive disposition* refers to the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics. If students are to develop conceptual understanding, procedural fluency, strategic competence, and adaptive reasoning abilities, they must believe that mathematics is understandable, not arbitrary; that, with diligent effort, it can be learned and used; and that they are capable of figuring it out.



Developing a productive disposition requires frequent opportunities to make sense of mathematics, to recognize the benefits of perseverance, and to experience the rewards of sense making in mathematics. A productive disposition develops when the other strands do and helps each of them develop. For example, as students build strategic competence in solving nonroutine problems, their attitudes and beliefs about themselves as mathematics learners become more positive. The more mathematical concepts they understand, the more sensible mathematics becomes. In contrast, when students are seldom given challenging mathematical problems to solve, they come to expect that memorizing rather than sense making paves the road to learning mathematics, and they begin to lose confidence in themselves as learners. Similarly, when students see themselves as capable of learning mathematics and using it to solve problems, they become able to develop further their procedural fluency or their adaptive reasoning abilities.

Students' disposition toward mathematics is a major factor in determining their educational success. Students who view their mathematical ability as fixed and test questions as measuring their ability rather than providing opportunities to learn are likely to avoid challenging problems and be easily discouraged by failure. Students who view ability as expandable in response to experience and training are more likely to seek out challenging situations and learn from them...

The teacher of mathematics plays a critical role in encouraging students to maintain positive attitudes toward mathematics. How a teacher views mathematics and its learning affects that teacher's teaching practice, which ultimately affects not only what the students learn but how they view themselves as mathematics learners. Teachers and students inevitably

negotiate among themselves the norms of conduct in the class, and when those norms allow students to be comfortable in doing mathematics and sharing their ideas with others, they see themselves as capable of understanding...Some of the most important consequences of students' failure to develop a productive disposition toward mathematics occur in high school, when they have the opportunity to avoid challenging mathematics courses.