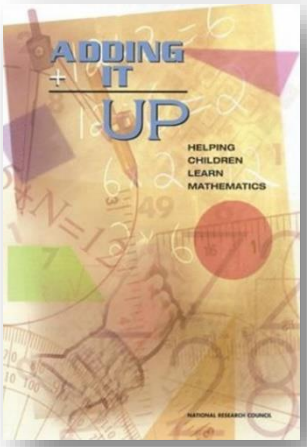
The Components of Mathematical Proficiency

Adaptive Reasoning



Adaptive reasoning refers to the capacity to think logically about the relationships among concepts and situations. Such reasoning is correct and valid, stems from careful consideration of alternatives, and includes knowledge of how to justify the conclusions. In mathematics, adaptive reasoning is the glue that holds everything together, the lodestar that guides learning. One uses it to navigate through the many facts, procedures, concepts, and solution methods and to see that they all fit together in some way, that they make sense. In mathematics, deductive reasoning is used to settle disputes and disagreements. Answers are right because they follow from some agreed upon assumptions through series of logical steps.

Students who disagree about a mathematical answer need not rely on checking with the teacher, collecting opinions from their classmates, or gathering data from outside the classroom. In principle, they need only check that their reasoning is valid. Many conceptions of mathematical reasoning have been confined to formal proof and other forms of deductive reasoning. Our notion of adaptive reasoning is much broader, including not only informal explanation and justification but also intuitive and inductive reasoning based on pattern, analogy, and metaphor…One manifestation of adaptive reasoning is the ability to justify one's work. We use *justify* in the sense of "provide sufficient reason for." Proof is a form of justification, but not all justifications are proofs. Proofs (both formal and informal) must be logically complete, but a justification may be more telegraphic, merely suggesting the source of the reasoning…

Classroom norms can be established in which students are expected to justify their mathematical claims and make them clear to others. Students need to be able to justify and explain ideas in order to make their reasoning clear, hone their reasoning skills, and improve their conceptual understanding. It is not sufficient to justify a procedure just once. As we discuss below, the development of proficiency occurs over an extended period of time. Students need to use new concepts and procedures for some time and to explain and justify them by relating them to concepts and procedures that they already understand. For example, it is not sufficient for students to do only practice problems on adding fractions after the procedure has been developed. If students are to understand the algorithm, they also need experience in explaining and justifying it themselves with many different problems.

Extracts from: National Research Council (2001) Adding it up: *Helping children learn mathematics.* J. Kilpatrick, J. Swafford, and B. Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.

Adaptive reasoning interacts with the other strands of proficiency, particularly during problem solving. Learners draw on their strategic competence to formulate and represent a problem, using heuristic approaches that may provide a solution strategy, but adaptive reasoning must take over when they are determining the legitimacy of a proposed strategy. Conceptual understanding provides metaphors and representations that can serve as a source of adaptive reasoning, which, taking into account the limitations of the representations, learners use to determine whether a solution is justifiable and then to justify it. Often a solution strategy will require fluent use of procedures for calculation, measurement, or display, but adaptive reasoning should be used to determine whether the procedure is appropriate. And while carrying out a solution plan, learners use their strategic competence to monitor their progress toward a solution and to generate alternative plans if the current plan seems ineffective. This approach both depends upon productive disposition and supports it.

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